Simultaneous astrometry of an image set : status report

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15/05/2015

Objectives

- Fitting astrometry of an image set using common objects (and possibly and external reference catalog)
  - Prior to stacking
  - Prior to simultaneous measurements on an image set (e.g. transient light curves)
  - Deriving an "instrument model"

Sketch

- Our images are already equipped with a catalog, and a rough WCS
- Stage 1 : associate all the catalogs
- Stage 2 : associate with an external position catalog, if needed (set the sideral frame)
- Stage 3 : fit simultaneously
  - Mappings from input image coordinates to some common frame ($\rightarrow$ WCS's)
  - Positions of the objects in common

Associating detections of the same object

The simple way :

- For each image
  - Load catalog and apply quality cuts
  - Match it to the “Object catalog” (in the tangent plane)
  - Add the unmatched objects to the “Object catalog”
- I know it is fast and efficient (with a 2D O(N1 log(N2)) matching)
- One could be worried by the outcome depending in principle of the order of input images.
- In practice, this is not serious if the WCS's are accurate enough (1-2 pixels) and blends are ignored (as they should).

Fit : Least Squares

$$\chi^2_{meas} = \sum_{c,d} [M_c(X_{c,d}) - P_c(F_i)]^T W_{c,d} [M_c(X_{c,d}) - P_c(F_i)]$$

Measurement terms

c,d : calexp, detection

M_c : mapping (pixel → TP), one per calexp

$X_{c,d}$: measured position of the object (pixels)

P_c : projection (sky → TP)

$W_{c,d}$: Measurement weight (1/var), transformed through M_c.

F_i : (sky) position of the object (measured as $X_{c,d}$)

$$\chi^2_{ref} = \sum_{j} [P(F_j) - P(R_j)]^T W_j [P(F_j) - P(R_j)]$$

Reference terms

P : some (user-provided) projection

F_j : (fitted) sky position of the object

R_j : sky position of the object fitted (reference catalog)

W_j : R_j weight (1/var), transformed through P

Least Squares (2)

$$\chi^2_{meas} = \sum_{c,d} [M_c(X_{c,d}) - P_c(F_i)]^T W_{c,d} [M_c(X_{c,d}) - P_c(F_i)]$$

Measurement terms

...

$W_{c,d}$: Measurement error, transformed through M_c.

…. → so W depends on some fitted parameters !

Yes, but in practice, the scale of M is extremely well known, so this can be ignored.

$$\chi^2_{ref} = \sum_{j} [P(F_j) - P(R_j)]^T W_j [P(F_j) - P(R_j)]$$

Reference terms

P : some (user-provided) projection

Why $P(F_j)-P(R_j)$ rather than just (F_j-R_j)?

because the distance on the sphere is **not** Euclidean

Least Squares (3)

$$\chi^2_{meas} = \sum_{c,d} [M_c(X_{c,d}) - P_c(F_i)]^T W_{c,d} [M_c(X_{c,d}) - P_c(F_i)]$$

- This setup accommodates the fit of mappings between images:
  - All the P_c are set to identity.
  - One of the M_c is set to identity.
  - No external catalog nor "reference terms".
- This yields the optimal mapping between images, given position measurements and their uncertainties.

Implementation

- Least squares with (mostly analytical) computation of derivatives (w.r.t positions and parameters).
- Sparse matrix algebra (Eigen3 package). Similar performance with Cholmod.
- About 1500 lines of new C++ code (~ 10 classes) to implement the fit and the model. Used existing (home-made) classes for everything else.
- The fit talks to the model via two abstract classes.

Outlier removal

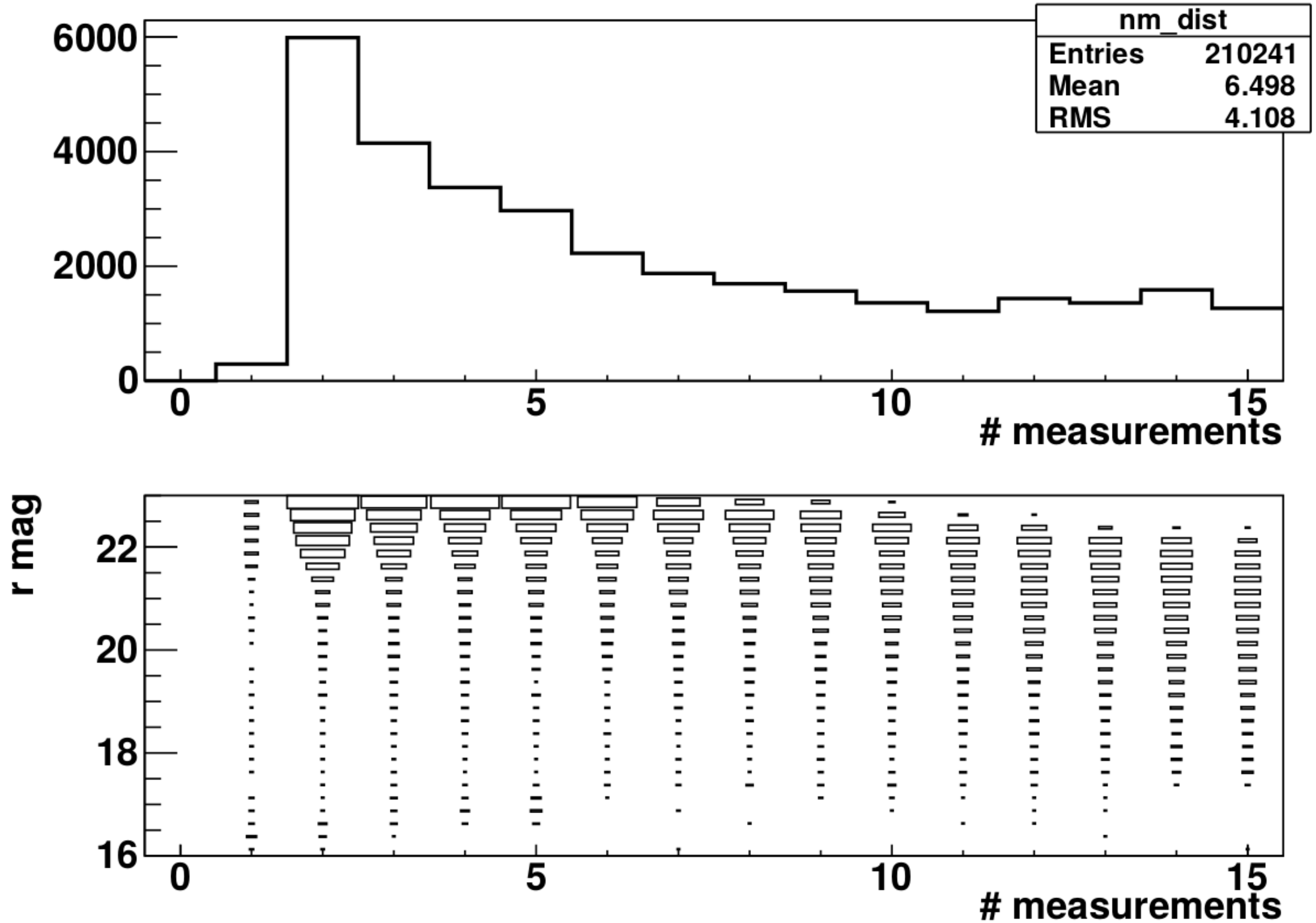
- I have not found a **canned** small-rank update of Cholesky factorizations. The only one I know about is Cholmod providing a rank-1 update.
- So I have used the following trick: do not remove in a single pass 2 outliers that constrain the same parameter.
- Would require a lot of iterations to come to zero outlier removed.
- I was not that patient: I ran it only 4 iterations.

Trial run

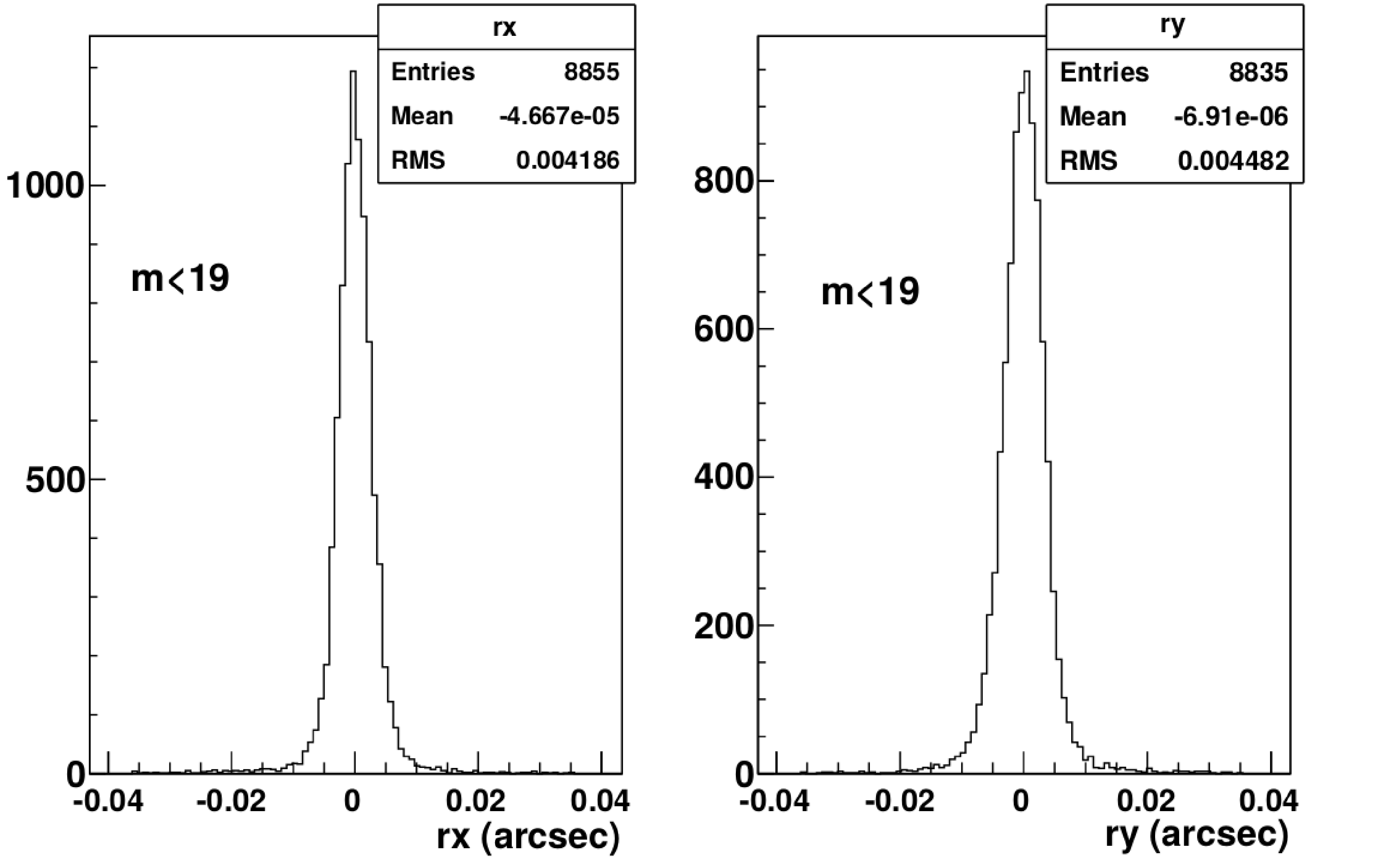
- 15 Megacam r-band 300-s exposures on D3, observed over the same lunation. 540 Calexp's.
- Use USNO-A2 for the reference catalog.
- Ignore proper motions.
- Use Gaussian-weighted positions and associated errors.
- Strict selection of measurements (no flag at all, S/N>10), average of ~400 measurements/calexp
- All calexps have their own mapping parameters as if they all came from different instruments

Trial run

- ~32,500 objects, ~210,000 measurements.



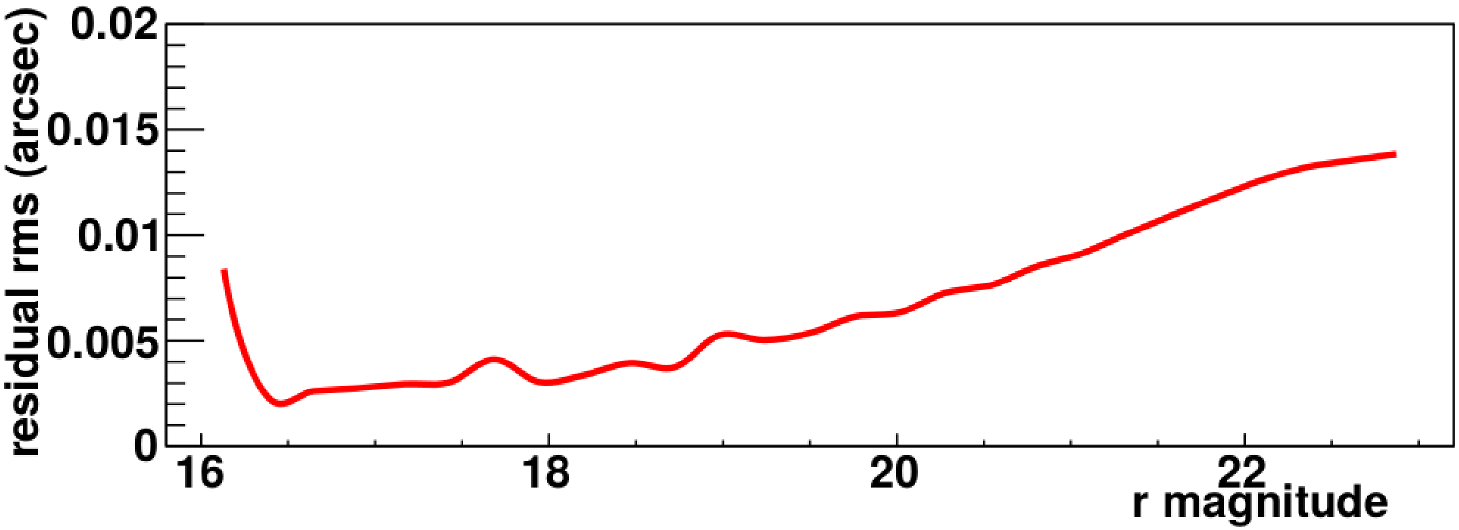
Trial run : residuals

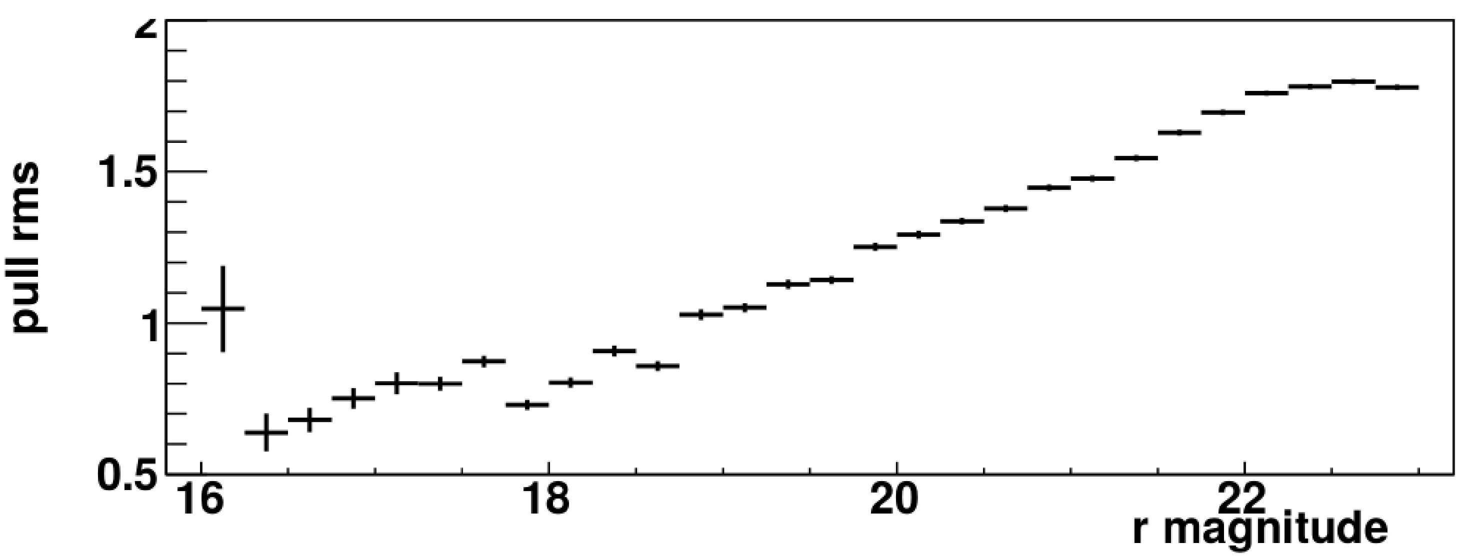


Residuals of the “measurement terms”, these are internal residuals

Residuals vs mag

Use simplistic error model $V = V_{meas} + (0.02\ \text{pix})^2$





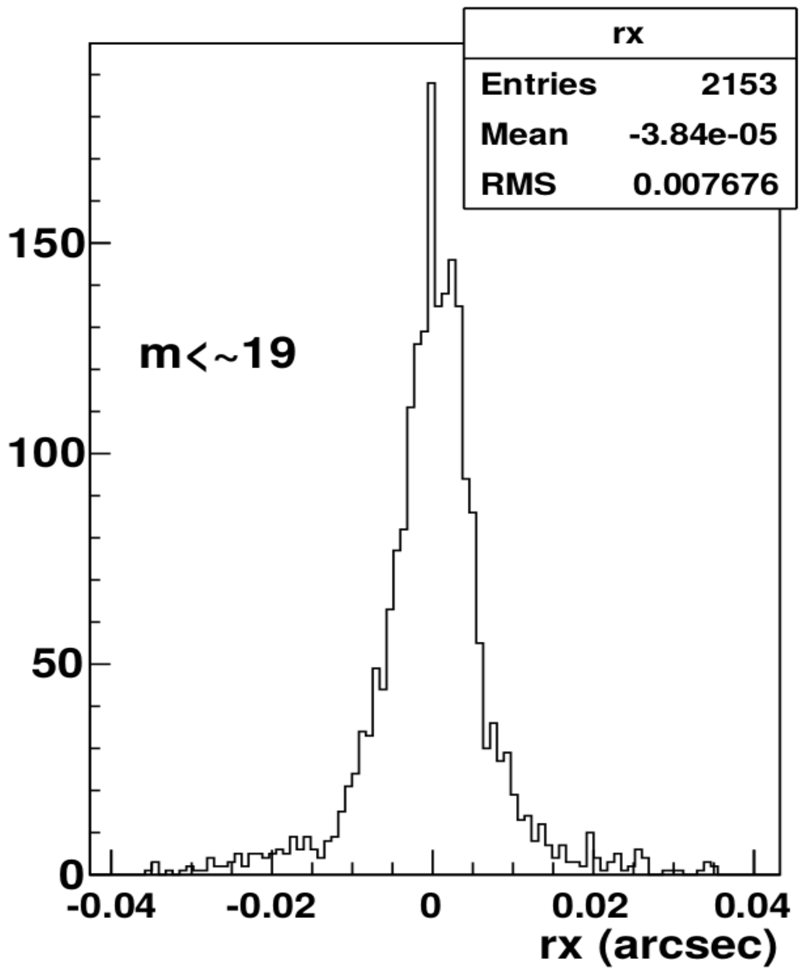
Beware : residuals are **not** Studentized, and The number of measurement is not that large

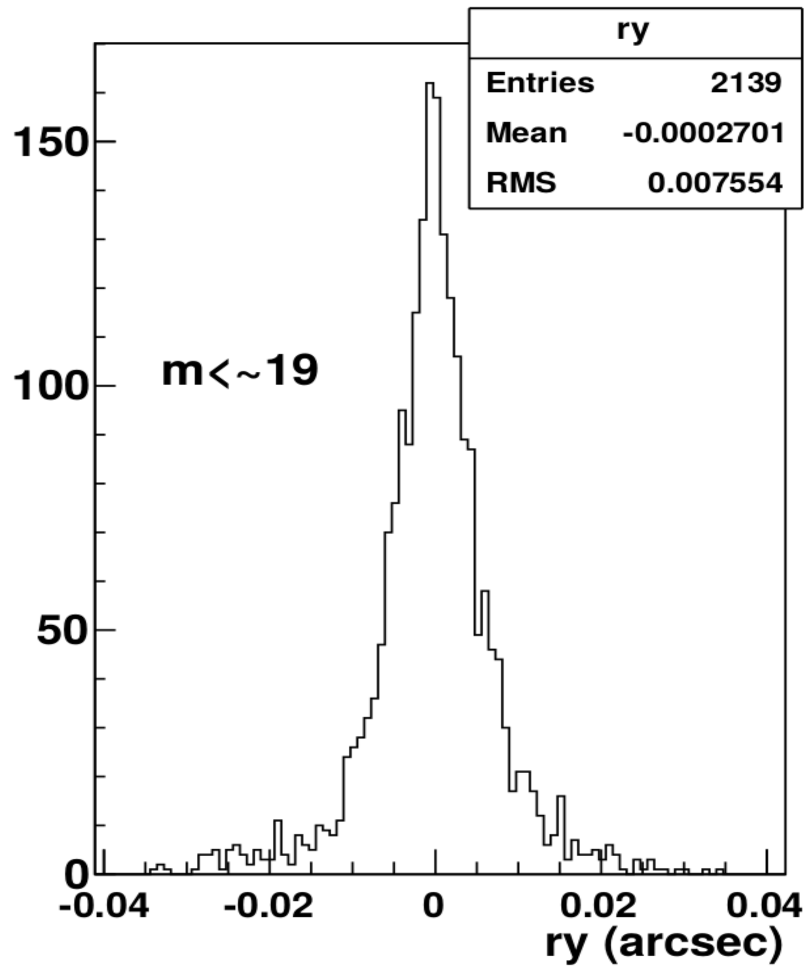
A smaller constant term would help for the bright side, but not for the faint side

Overall : it could be much worse!

Second trial run: Suprime cam

120-s (i and z)-band exposures reduced by Augustin Guyonnet.
12 exposures, guessed photometric scale. Exactly the same code.





Computer resources

- Execution time: for the 15 Megacam exposures:
  - Reading the (540) catalogs : ~ 100 seconds at 33% CPU
  - Associating : negligible
  - Fitting: <~ 20 s per iteration
    - Computing the derivatives : ~ 1 s
    - "Squaring" the Jacobian, i.e. $H = JJ^T$: ~ 3s
    - Factorize-solve-update (dim=75,510, nnz=17,164,700) ~13 s.
  - Partial fits (positions OR mappings) are solved instantly.
  - Total : 125.137u 19.769s 3:40.44 65.7% (Xeon 2.3 GHz)
- Memory reaches ~1GByte (not completely sure though)

To be done (at least):

- I still have to code the mapping model for a “rigid instrument”. I would not be surprised if the residuals come out much larger. Have to think about relaxing rigidity.
- Study dispersions of faint objects (galaxies mostly).
- Proper motions, parallaxes ? the code to handle proper motions is there, but I have not implemented anything to detect “moving” stars.
- Output of mappings (i.e. WCS's)... Which format? Output of the catalog.

(Simplified) class diagram