Aspects of Cosmology with Scalar Fields[1]

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Abstract

In the last few years there has been considerable interest, primarily due to observations of distant supernovae, in the possibility that a significant fraction of the energy of the Universe today is in a coherent zero mode of a very weakly coupled scalar field. This possibility is just one very specific way in which such scalar fields could be important cosmologically. We first review briefly the observational constraints which exist on the possible contributions to the energy density of the Universe as a function of redshift/temperature, and then discuss the potentials which are associated with simple scalings of the scalar energy density. We outline three different modifications of the standard radiation and matter FRW cosmologies: a pre-nucleosynthesis phase dominated by the kinetic energy of a scalar field ("kination"), the addition of a scalar field component mimicking the dominant radiation or matter ("self-tuning" scalar field cosmology), and a phase of inflation type behaviour today ("quintessence"). We discuss how each scenario can be realised in a scalar field potential, focussing on the question of fine-tuning in each case. We emphasize how the problem of fine-tuning can often be moved between the initial conditions and the potential, and that care should be taken in assuming one form preferable over the other. For quintessence we write down simple criteria for a potential to have the desired properties while avoiding fine-tuning of initial conditions, and discuss the role attractor and "tracking" solutions play in this context. Finally we discuss a scenario in which the Universe is reheated by gravitational particle production. Kination is a necessary part of such a model, and it also provides a framework in which the early Universe inflaton can be the same field driving acceleration today. Constraints for Particle Physics model building on these scenarios are briefly discussed.

[1]This paper is the "Habilitation" thesis of the author. To refer to it please cite it as "M. Joyce, Habilitation thesis, Université Paris XI (2001)".

1 Introduction

Until a couple of years ago 'scalar fields in cosmology' could almost have been a synonym for inflation. The focus of this brief review is precisely the role scalar fields can play - in particular homogeneous modes of scalar fields - in modifying the standard FRW phase which describes the more observationally accessible ("post-inflationary") realms of cosmology. Without doubt the great part of the interest in recent years in such possibilities has been motivated by the indications from observations of Supernovae that the Universe is now accelerating [1, 2]. Explained in terms of a scalar field component, this corresponds essentially to inflation, but at a much lower energy scale (that characteristic of the energy density in the Universe today) to that envisaged in standard inflation. The approach we take here is to consider this as just one of the many possible roles of scalar fields in post-inflationary cosmology left open by observational constraints on the standard FRW model. This approach reveals that there are interesting and quite rich possibilities that are still open which are not of the inflationary type. In particular we discuss the possibility and implications of domination by the kinetic energy of a scalar field before nucleosynthesis ("kination"), and of attractor solutions in which the scalar field energy scales in the same way as the energy density of the component of radiation or matter ("self-tuning" cosmology). This more general consideration of scalar field dynamics also throws light on the problems of fine-tuning, which are of particular relevance to scenarios involving a phase of inflation beginning today (so-called "quintessence"). We discuss in particular at some length the necessary features of a potential in order that it produce acceleration today without fine-tuning of initial conditions. We also describe an alternative scenario for exit from inflation to the radiation dominated FRW phase which requires a phase of "kination", and can potentially lead to the identification of the inflaton with the field driving acceleration today.

While we try to focus as much as possible on the observational implications or signatures of these models, the speculative nature (at least for the present) of these constructions should not be lost from view. We still have, even in terrestial experiments, no convincing observational evidence for the existence of scalar fields as constituents of the physical world! Further one should not forget that observational cosmology is not like laboratory or accelerator experimental physics: there are much greater uncertainties and unknowns in observations of the Universe than in terrestrial experiments, tied to the enormous intrinsic limitations on our capacity to choose what we observe. Supernovae results are a good case in point. These results have been the main impetus for recent work on scalar field cosmology. Yet it is easy to imagine that currently unknown systematics will turn out to be at the origin of the very slight dimming of supernovae as a function of their cosmological distance, and indeed some such possible effects have already been discussed in the astrophysical literature on these results [3]. From a theoretical point of view therefore it makes sense to concentrate on the essential elements of these scenarios rather than to enter too far into the details. One of the central points of this small review is in fact to place theoretical considerations associated with these latter observations in the wider context of cosmology with scalar fields. Even if such results thus ultimately turn out to have been incorrect, we may hope not have been totally wasting out time! Scalar fields do after all form an essential element of all current Particle Physics models. Because of their defining (trivial) transformation properties under coordinate transformations, they can have a condensate state which is compatible with the symmetries of the Friedmann Roberston Walker(FRW) solutions i.e. by taking a non-zero expectation value for a scalar field

$$\langle \phi \rangle \neq 0 \tag{1}$$

we do not necessarily break the isotropy and homogeneity underlying the FRW solutions. Indeed what we are doing in this review is considering in a quite general manner the possible effects of precisely those condensate states which can be described by exact FRW solutions. Our approach is to assume initially we know nothing about the (effective) potential governing the evolution of this condensate and to see what relatively simple variants of the standard FRW cosmology may be interesting to consider from the point of view of their cosmological implications.

The structure of the review is as follows. We first recall in section 2 the basics of the dynamics of homogeneous scalar fields in a FRW Universe, noting in particular how the cosmological scaling properties of the energy density depend on the ratio of kinetic to potential energy in the scalar field. In section 3 we derive explicitly the potentials which give simple behaviours of the scalar field energy density, and explain in particular the interest of simple exponential potentials. We then consider in section 4 the observational constraints that exist on an energy density in a scalar field allowed to have any scaling consistent with this scalar field dynamics. In section 5 we outline the three variants on standard FRW cosmology which we choose to study further: a radically modified cosmology prior to nucleosynthesis in which a kinetic mode of a scalar field dominates at this time; a cosmology modified in the era of structure formation by a scalar field component which adapts its scaling to that of the dominant component; and the recently much investigated possibility that a scalar field dominates the Universe today in an accelerating phase. In section 6 we focus on the potentials and dynamics for the scalar field required in each of these scenarios, highlighting the relation between the fine-tuning of model parameters and that of initial conditions. In the following section we discuss at further length the particular case of quintessence, describing general criteria for a model without fine-tuning of initial conditions in the early Universe, and which also put the "coincidence problem" in these models in a very evident form. In section 8 we describe an alternative scenario for how a primordial inflationary phase is matched onto the radiation dominated FRW Universe, in which the interpolating phase is precisely the first kind of model we discussed ("kination"). We describe briefly how this model can have a modification in which the inflaton becomes also the field driving quintessence. In section 9 we review briefly the main features of the models we have described in relation to the question of what their possible origin in Particle Physics might be. Finally, before concluding with some remarks on the outlook for observational constraint on these models, we briefly discuss some variants on the framework we have worked in which try to address in particular the "coincidence" problem of quintessence models.

2 Basics of Homogeneous Scalar Fields

In this section we review the essentials of the dynamics of homogeneous modes of scalar fields in the FRW Universe.

2.1 Equations of Motion

We consider first, and throughout most of this review, a *real* scalar field *minimally coupled to gravity*, and completely *uncoupled to ordinary (baryonic or dark) matter* (i.e. sufficiently weakly coupled that such coupling can be neglected). The generalisation to the case of complex scalar fields is trivial (being equivalent to just two real scalar fields). In section 9 we will discuss why the coupling to visible matter in particular is extremely tightly constrained, while in section 10 we will briefly discuss some proposed models in which some of the other assumptions are relaxed.

The action of the scalar field is thus

$$\mathcal{S}_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{2}$$

where $V(\phi)$ is its (effective) potential, and its energy momentum tensor is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right] \tag{3}$$

which can be recast in the form

$$T_{\mu\nu} = (p+\rho)\epsilon U_\mu U_\nu - p g_{\mu\nu} \tag{4}$$

with the identifications

$$p = \frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi) \qquad \rho = \frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi) \tag{5}$$

$$U_\mu = \frac{\partial_\mu\phi}{\sqrt{|g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi|}} \qquad \epsilon = sgn\left(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right) \tag{6}$$

The homogeneity and isotropy of the FRW solutions of Einstein's equations require that the scalar field be a function of time only i.e. $\phi \equiv \phi(t)$ where t is the cosmic time in which the FRW metric has the form $ds^2 = dt^2 - a^2(t)ds_3^2$ where ds_3 is the metric on the homogeneous and isotropic spatial three surfaces.

In this case the energy momentum tensor corresponds to that of an ideal fluid (with $U_\mu = (1,0,0,0)$ corresponding to the fluid at rest in the comoving coordinates), with

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{7}$$

The equation of motion for ϕ is

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) + \frac{\partial V}{\partial\phi} = 0 \tag{8}$$

which, for the homogeneous mode in the cosmology FRW, gives simply

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,. \tag{9}$$

where $V'(\phi) = dV/d\phi$, and the expansion rate $H = \frac{\dot{a}}{a}$ is given by the Einstein equation

$$H^2 = \frac{1}{3M_P^2}\left(\rho_n + \rho_\phi - \frac{k}{a^2}\right). \tag{10}$$

Here k is the integration constant which fixes the curvature of spatial sections in the FRW model, which we will assume to be zero, corresponding to a flat Universe, unless stated otherwise explicitly. $M_P = (1/\sqrt{8\pi G}) \approx 2.4\times10^{18}$GeV is the *reduced* Planck mass which we will use throughout below. Finally ρ_n is the (homogeneous and isotropic) energy density in the other (uncoupled) components of matter or radiation which we assume to make up the energy in the Universe, and which we take to have a fixed scaling as a function of red-shift $\rho_n \propto 1/a^n$ (thus $n=3$ for matter, $n=4$ for radiation). This scaling is inferred from the conservation of their energy momenta, for which the perfect fluid form yields

$$\dot{\rho} + 3H(p+\rho) = 0 \tag{11}$$

with $n = 3(w+1)$ where $p = w\rho$ is the appropriate equation of state ($w=0$ for matter, $w=1/3$ for radiation).

With the scaling of ρ_n specified, Eqs. (9) and (10) form a closed set of equations for the evolution of the FRW cosmology, with boundary conditions on ϕ,$\dot{\phi}$ and ρ_n (provided H is always positive definite, which we assume to be the case). It is useful also to write explicitly the equation for the acceleration of the Universe (which can be derived from these equations) which is

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}\left[\rho_n(n-2) + \rho_\phi + 3p_\phi\right]. \tag{12}$$

2.2 Scaling of energy density in the scalar field

On multiplication by $\dot{\phi}$ equation (9) can be written as

$$\frac{d}{dt}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] + 3H\dot{\phi}^2 = 0. \tag{13}$$

Defining ξ as the *ratio of the kinetic energy to the total scalar energy* i.e.

$$\xi(t) = \frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi(t))} = \frac{\frac{1}{2}\dot{\phi}^2}{\rho_\phi(t)} \tag{14}$$

we find

$$\rho_\phi(t) = \rho_\phi(t_o)\exp\left[-\int_{t_o}^{t} 6\xi(t)H(t)dt\right] = \rho_\phi(a_o)\exp\left[-\int_{a_o}^{a} 6\xi(a)\frac{da}{a}\right]. \tag{15}$$

Note that here no specific assumption has been made about $H = \dot{a}/a$. It is linked by the Einstein equation (10) to the *total* energy density. How the energy density in the scalar sector depends on the scale factor a is determined solely by the function $\xi(a)$ i.e. by how its energy is distributed between its kinetic and potential energy. That this is so is in fact obvious when we treat the scalar field condensate as a perfect fluid, for which the scaling of the energy density is manifestly determined once its equation of state is specified from the conservation of energy momentum equation (11), which is identical to (13) since from (7) it follows that the equation of state of the scalar field is

$$p = w\rho \qquad w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi(t))}{\frac{1}{2}\dot{\phi}^2 + V(\phi(t))} \equiv 2\xi - 1. \tag{16}$$

We will often work below by choice with the variable ξ, rather than w, because its simplicity as a direct ratio of energies makes its interpretation more immediately transparent in the case of a scalar field.

In differential form we have

$$\xi(a) = -\frac{1}{6}\frac{a}{\rho}\frac{d\rho}{da}. \tag{17}$$

Assuming that the potential is *non-negative* we have that $0 \le \xi \le 1$, and therefore

$$0 \le -\frac{a}{\rho}\frac{d\rho}{da} \le 6. \tag{18}$$

When ξ is a constant this corresponds to

$$\frac{\rho(a)}{\rho(a_o)} = \left(\frac{a_o}{a}\right)^n \qquad 0 \le n \le 6 \tag{19}$$

with $n = 6\xi$. In the limit that $\xi \to 1$ i.e. that the kinetic energy of the scalar field is very dominant over its potential energy, we have the most rapid possible redshifting of the total energy as $1/a^6$, while the opposite limit $\xi \to 0$ i.e. of potential energy dominance corresponds to a constant energy density.

If we assume that the scalar energy density dominates the total energy density i.e. $\rho_\phi >> \rho_n$ (and we neglect the curvature term), it is easy to calculate for constant ξ the behaviour in time of the scale factor to find

$$a \propto t^{1/3\xi} \tag{20}$$

for $\xi \neq 0$. For $\xi < 1/3$ one therefore has familiar superluminal i.e. inflationary type dynamics (with the limit $\xi \to 0$ approaching pure exponential inflation). For $\xi > 1/3$ we have a phase of subluminal expansion.

When $\xi > 2/3$ such a phase corresponds to a red-shifting of the energy density more rapid than that in radiation. In line with the nomenclature introduced in [4] we will refer to such a phase as *kination*, since it corresponds to a dominance of the total energy density by the kinetic energy of the scalar field .

To understand the origin of the limiting $1/a^6$ scaling it is perhaps useful to note that whenever the $V'(\phi)$ term is zero, or is small enough that it can be neglected in (9), (i.e. the field is overdamped with $3H\dot{\phi} >> V'(\phi)$) we can integrate directly to get

$$\frac{1}{2}\dot{\phi}^2 \propto \frac{1}{a^6} \tag{21}$$

The exact $1/a^6$ limit will be attained in the case that the potential is exactly flat, which corresponds to a symmetry in the potential. The scaling can then be understood in terms of the conservation of the corresponding conserved charge, the charge being proportional to the comoving volume of space and the time derivative of the field in this direction i.e. $Q \propto a^3\dot{\phi} =$ constant.

Note that the scaling of the kinetic energy in Eq. (21) is not simply a special case of the result for the scaling of the total energy density. It requires only that the gradient term be negligible, not that $\xi \to 1$. When $\xi \to 1$ the potential, and also its gradient, become negligible, and therefore $\rho_\phi \to \frac{1}{2}\dot{\phi}^2 \propto 1/a^6$. However if $\xi \to 0$ so that the potential dominates the total energy we can still have that a sufficiently flat potential that Eq. (21) holds. For example this will describe the case of entry to an inflationary solution in a very flat potential, which is extremely efficient precisely because the kinetic energy scales away rapidly as given by Eq. (21), driving us closer to $\xi = 0$ i.e. into a rapidly inflating solution.

2.3 Potential Reconstruction

What the scaling which actually results from a given potential is must be determined by solving the coupled set of equations (9) and (10), together with the initial conditions on ϕ and $\dot{\phi}$ and ρ_n. Considering the inverse problem, it is not difficult to see that, in principle, for any desired behaviour - consistent with equation (18) - one can design a potential and initial conditions on the scalar field to produce it: We suppose that we are given continuous functions $\rho_\phi(a)$ and $\rho_n(a)$ explicitly in some range of red-shift $[a_1, a_2]$. First, from $\rho_\phi(a)$ we can calculate $\xi(a)$ through Eq. (17) which suffices in Eq. (14) to determine $\dot{\phi}^2(a)$ and $V(a)$. Taking some initial condition on ϕ (say $\phi(a_1) = 0$), and definite choice of sign for $\dot{\phi}$ (say > 0), one can find, provided $\dot{\phi} \neq 0$, a monotonic solution for $\phi(a)$ in the range $[a_1, a_2]$, from $\frac{d\phi}{da} = \dot{\phi}(a)/Ha$ where H is given by the Einstein equation (10). Inverting this solution for $\phi(a)$ to derive a as a function of ϕ, $V(a)$ then gives the required potential $V(\phi)$. We can relax the condition that $\dot{\phi} \neq 0$ to treat the case where $\dot{\phi} = 0$ on a set of points of measure zero by appropriate matching of the solutions in the connected ranges in which $\dot{\phi} \neq 0$. The only case which cannot be modelled by a potential is a phase of exact exponential inflation (with $\dot{\phi} = 0$ for some of the range of red-shift) matched onto some other scaling: If the field doesn't roll it cannot move out of the region it is in! This is just the well known fact that to exit from inflation we need some curvature on the potential (which always leads to some deviation form pure 'scale-invariance' in the spectrum of perturbations).

A simple example of this (non-unique) inversion process will be given below in section 3 where we analyse what potentials are associated with simple behaviours of the scalar energy density. In particular we will see that for simple exponential potentials there exist attractor solutions which give a simple way of realizing any desired scaling at fixed n. For the moment we disregard any consideration of what kind of potential may be realistic (or at least plausible) from Particle Physics considerations, and treat the scalar field energy as one which gives an a priori unknown contribution to the energy of the Universe as a function of redshift.

3 Potentials for simple scaling behaviours

We now turn to the question of what kinds of potentials (and ultimately initial conditions) for scalar fields one requires in order to realize a cosmological history involving a significant role for scalar field energy. To do so we first return to the general inversion procedure described in section 2.3, and implement it in various simple cases.

3.1 Exact fixed scaling solutions in exponential potentials

We consider the case of a constant scaling law i.e. ξ constant. We can then express the kinetic and potential energy as a function of a and the potential energy at some time $V(\phi_o)$ as

$$\frac{1}{2}\dot{\phi}^2 = \frac{\xi}{1-\xi} V(\phi_o) \left(\frac{a_o}{a}\right)^{6\xi} \qquad V(\phi) = V(\phi_o) \left(\frac{a_o}{a}\right)^{6\xi} . \tag{22}$$

for $\xi \neq 1$. (For $\xi = 1$ the solution is trivially $V(\phi) = 0$).

Let us first consider the case that the energy in the scalar field dominates the energy density. Setting $\rho_n = 0$ in the Einstein equation we find that

$$\frac{d\phi}{da} = \frac{\dot{\phi}}{aH(a)} = \sqrt{\frac{3\xi}{4\pi G}} \frac{1}{a} \tag{23}$$

which when integrated and used in the expression above for the potential gives

$$V(\phi) = V(\phi_o) e^{-\sqrt{48\xi\pi G}(\phi - \phi_o)} \equiv V_o e^{-\sqrt{6\xi}\frac{\phi}{M_P}} \tag{24}$$

where $M_P = \sqrt{8\pi G}$ is the reduced Planck mass. Note that here, as $\dot{\phi} \neq 0$, there is actually no arbitrariness in the inversion. This means that an exponential potential is the only way to produce an exactly constant scaling (in the assumed limit of scalar field domination). We will see below that an almost constant scaling can be found in various other cases as well.

Why one obtains different scalings as a function of the slope of the exponential can be easily understood intuitively: The dynamics of the scalar field is simply a damped roll, and the scaling depends, as we described in section 2, on ξ i.e. on the relative importance of kinetic and potential energy. In a very steep potential the gain in kinetic energy due to the roll down is greater than the loss due to the damping, so that the kinetic energy dominates. In a flat potential the damping wins and ξ becomes small. Exponentials are simply the potentials in which the relevant ratio V'/V is constant. Thus one can use exponentials as a kind of 'yard-stick': A region of a potential flatter than an exponential with $\lambda = 1$ i.e. for which

$$\lambda_{eff} = \frac{M_P V'}{V} < 1 \tag{25}$$

will tend to produce a scaling corresponding to superluminal solutions, while ones with $\lambda_{eff} > 1$ will tend to produce subluminal scaling, with in particular those with $\lambda_{eff} > 2$ tending to produce kination type behaviour. This is a familiar kind of condition encountered in standard inflationary models[2][18].

These exponential potentials have another important property: a simple dynamical analysis [19, 20, 21] of them shows that, for $\lambda < \sqrt{6}$ they are attractor solutions to the system of coupled equations (for scalar

[2]In this context a condition like (25) is usually given together with a similar one on the second derivative, to get the full 'slow-roll' conditions, which correspond to requiring that the $\ddot{\phi}$ term be completely negligible in the equation of motion of the scalar field.

field only). This will be important in the discussion which follows on initial conditions for cosmological models. For $\lambda > \sqrt{6}$ there is no attractor solution, but the behaviour is simple: for these values the potential is sufficiently steep that the potential energy is always asymptotically negligible and the scaling approaches $\rho_\phi \propto 1/a^6$ with arbitrary accuracy, corresponding to $\xi = 1$. Note thus that, perhaps counterintuitively at first, this limit can be associated with a very steep direction in a potential, or with a direction with no potential at all[3].

So far our considerations were for the case of scalar field domination, which was assumed in arriving at equation (23). We can also easily find solutions in which the total energy density scales as like that the other component $\rho_{total} \propto \rho_n \propto 1/a^n$. Then in place of equation (23) we have

$$\frac{d\phi}{da} = \frac{\dot{\phi}}{aH(a)} = \sqrt{\frac{3\xi}{4\pi G}\frac{V_o}{\rho_o(1-\xi)}}\; a^{\frac{n}{2}-3\xi-1} \tag{26}$$

where ρ_o is the total energy density at $a = a_o = 1$. For the particular case $n = 6\xi$ i.e. in which we require the scalar field energy to have the same scaling as the total energy density we again find the solution to be an exponential potential

$$V(\phi) = V_o e^{-\frac{\sqrt{n}V_o}{\rho_o(1-\xi)}\frac{\phi}{M_P}} \equiv V_o e^{-\lambda\frac{\phi}{M_P}} \tag{27}$$

(where $V_o/(1-\xi)$ is the energy density in the scalar field). Since the total energy density is the sum of the two components, we have that the exponent in the exponential $\lambda \geq \sqrt{n}$. We can also express the contribution of the energy densities in the two components in terms of the exponent as

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{total}} = \frac{V_o}{\rho_o(1-\xi)} = \frac{n}{\lambda^2} \qquad \Omega_n = 1 - \Omega_\phi \tag{28}$$

and write the solution for the scalar field explicitly as

$$\phi(t) = \frac{2M_P}{\lambda}\ln\frac{t}{t_o} \tag{29}$$

where t_o is the particular time defined by $V(t = t_o) = V_o$. It has been shown [22, 23, 21] also in this case that this solution is an attractor for $\lambda \geq \sqrt{n}$ to the coupled set of equations, with both scalar field and the other component. It is easy to understand qualitatively the origin of the condition $\lambda > \sqrt{n}$: in this case the first attractor solution discussed above, which was for the case of pure scalar field, gives (for $0 < \lambda^2 = 6\xi < 6$) a scaling $a^{-\lambda^2}$. If the scalar field dominates at any time over the other component this solution will approximately describe the evolution. For $\lambda > \sqrt{n}$ this scaling in this solution will always tend to make the scalar field energy decrease back towards the energy density in the other component; on the other hand, if the scalar field energy continues always to scale this way it will become very sub-dominant, and strongly damped, therefore scaling slower than the dominant component, thus making its energy density again approach that of the latter. The attractor solution represents a 'compromise' between the two regimes which is just such as to make the scalar field scale in the same way as the other component. For the case $\lambda < \sqrt{n}$, on the other hand, since again the additional component can only make the scalar field energy scale slower than in the pure scalar field solution, and therefore the scalar field is driven always asymptotically to complete dominance in the first attractor solution.

3.2 Approximate scaling solutions in inverse power law potentials

Looking further at more general solutions [23] of equation (26) for the case $n \neq 6\xi$, to describe the case of constant scaling in the scalar field energy when the other component dominates the energy density, we

[3]See section 6.2 for a more detailed analysis which shows why $\lambda = \sqrt{6}$ is the critical value.

find, after integration and the inversion of $\phi(a)$ the potential

$$V(\phi) = V_o \left[\frac{n-6\xi}{2\phi'_o a_o}(\phi - \phi_o) + 1 \right]^{\frac{1}{2}\frac{6\xi}{6\xi - n}} \tag{30}$$

where $\phi'_o = d\phi/da(a = a_o)$. Choosing the initial condition $(n - 6\xi)\phi_o = 2\phi'_o a_o$ the solution becomes a simple power law potential in ϕ. For $n < 6\xi$ the power is positive; this describes the case of a sub-dominant scalar energy which decreases faster than the dominating component, and is therefore not of particular interest in the present context. The case $n > 6\xi$, corresponding to an *inverse* power law, does however describe a case of potential cosmological interest: a scalar field energy evolving in this solution will always eventually become a significant contribution to the total energy density. The solution for the field (with the chosen initial conditions) describes the roll away to large values of the field

$$\phi(a) = \phi_o \left(\frac{a}{a_o} \right)^{\frac{(n-6\xi)}{2}} . \tag{31}$$

Writing the potential in the form $V(\phi) \propto \phi^{-m}$ we have the scalings

$$\rho_\phi \propto a^{-\frac{nm}{2+m}} \qquad \phi \propto a^{\frac{n}{2+m}} \tag{32}$$

As one would expect for the flat limit $m \to 0$ one recovers a pure cosmolgical constant type scaling, while for large m, when the potential is steep, one finds a behaviour like that of the exponentials with $\lambda > \sqrt{n}$ in which the scaling is essentially (but not exactly in this case) that of the dominant component.

In the literature on quintessence it is often stated that this latter solution is an attractor solution. This is not true strictly speaking, and it is important for the discussion which follows that we make the distinction between this case and the previous one. In the case of the exponential the solution is a true attractor in the sense that from *any* initial conditions on the system (specified by ρ_o, ϕ and $\dot{\phi}$) the asymptotic solution will be that given in equation (29) (which is of course explicitly independent of the initial conditions). In the present case the asymptotic solution to the evolution of the set of coupled equations is certainly not given by equation (31), as the approximation that $\rho_n >> \rho_\phi$ under which the solution is valid always breaks down at some finite time. The solution represents in fact only a transcient behaviour on the way to the true asymptotic attractor which is a scalar dominated inflationary solution at large values of the field. As we will discuss further in section 7 for a certain basin of initial conditions this solution does describe the transcient behaviour, and in that case they can be said to show attractor-like behaviour in the sense that, for this basin of initial conditions, ϕ and $\dot{\phi}$ will go through a period in which (32) holds true, giving a scaling solution for the scalar field energy density. Further the dominant energy density ρ_n has a determined evolution in these solutions so that its density is determined at the time of transit to the asymptotic inflationary attractor.

3.3 Approximate constant scaling in an oscillatory mode

Another case in which one can find [24] a simple scaling behaviour for the scalar field energy is for a field oscillating about a minimum of its potential. In this case the equation of motion of the scalar field is just that of a damped oscillator. An oscillation can occur when the (intrinsic) period of the undamped oscillator is greater than the age of the Universe i.e. when the frequency $H < \omega$. As H decreases the system inevitably oscillates, and given that its period is short compared to the expansion rate, one can replace ξ in Eq. (15) by its value $\langle \xi \rangle$ averaged over a period to find the scaling behaviour of the total energy density. For a simple quadratic potential we have the simple equipartition of energy of the harmonic oscillator $\langle \frac{1}{2}\dot{\phi}^2 \rangle = \langle V(\phi) \rangle$ so that $\langle \xi \rangle = 1/2$ and therefore $\rho \propto 1/a^3$. In fact for the more general case of a polynomial form in the potential ϕ^n (n an even integer) it is possible to solve exactly to find

$$\langle \xi \rangle = \frac{n}{n+2} \tag{33}$$

and therefore

$$\rho_\phi \propto a^{-\frac{6n}{n+2}}. \tag{34}$$

Note that for $n \to \infty$ we attain again the limit of $1/a^6$ scaling: this is again due to the steepness of the potential which means that the kinetic energy dominates.

Note also an important difference with respect to the case of the rolling scalar field: the scaling does not in any way depend on what is driving the expansion. We only require $\omega >> H$, and once this is satisfied the scaling will hold for all subsequent evolution. It can change only because the potential changes form. For example, if we have a potential

$$V(\phi) = \lambda_2 \left(\frac{\phi}{M}\right)^2 + \lambda_4 \left(\frac{\phi}{M}\right)^4 + ... + \lambda_n \left(\frac{\phi}{M}\right)^n \qquad (n \text{ even}) \tag{35}$$

one can infer the approximate behaviour of the energy density as a function of red-shift, given the amplitude of the oscillation, as it is the dominant term which determines the scaling at that time.

4 Observational Constraints on Scalar Field Energy

Until the observations on supernovae which we will discuss below, the FRW cosmology considered as "standard" by most cosmologists was one in which the energy density was that associated only with matter and radiation. The idea that a homogeneous scalar field energy could be important was essentially limited to the context of inflation before the beginning of the FRW phase. As we will now discuss this (implicit) assumption that scalar fields do not play a cosmologically significant role after inflation is not at all one imposed by observational constraints, but just a simplification. Once one broadens one's interest to the class of FRW models including an unknown contribution from scalar fields, one finds that there is much space left for such modifications.

To quantify what the constraints are on homogeneous scalar field energy we consider the case of a FRW model with Einstein equation given by (10) where ρ_n represents the usual components of matter and radiation, and ask what the bounds are on $\rho_\phi(a)$.

4.1 The Universe at recent epochs

For any given (homogeneous FRW) cosmology, specified in terms of how the expansion rate varies as a function of red-shift, one can infer the distance (in some appropriately specified sense) as a function of red-shift. The most direct constraint we have on the energy content of the Universe is inferred by proceeding in the opposite direction - from the observed distance red-shift relation to constraints on the cosmology. The limitation on this very direct method is of course that distance measurement becomes extremely difficult at all but the very lowest ($z \leq 0.5$) redshifts. What astronomers do in practice is measure the flux received from an object (quantified by the "apparent magnitude" m) and its red-shift. The two are related to the real flux (quantified by the "absolute magnitude" M) via the relation

$$m = M + 5 \log d_L + 25 \tag{36}$$

where $d_L(z)$ is the *luminosity distance* of the object. If we know M, or can somehow infer it from some other measurable quantities, we can infer d_L. This is the method of "standard candles". The problem practically is that there are very few objects whose absolute brightness we know with much precision, in particular as we go to deeper red-shifts ($z \geq 1$). The observations on type IA supernovae reported in the last few years [1, 2] are the culmination of a long recognised potential for these astrophysical objects to act as standard candles: they are extremely bright (outshining a typical galaxy) and their intrinsic luminosity can be reliably reconstructed from measures of their decay time and spectra. The difficulty lies in the fact that they are rare and short-lived events (lasting a few weeks) but sophisticated observational techniques have been developed to observe enough of them (now about eighty in the useful red-shift range above $z \sim 0.3$ where the sensitivity to cosmology becomes more significant) to have a reasonable statistical sample.

The luminosity distance is related by $d_L(z) = d_M(z)(1+z)$ to the comoving distance d_M which is given by

$$d_M(z) = \frac{c}{H_o}\int_0^z \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\phi f(z)}} \qquad \Omega_m + \Omega_\phi = 1 \tag{37}$$

$$= \frac{c}{H_o}\frac{1}{\sqrt{\Omega_k}}\sinh\left[\sqrt{\Omega_k}\int_0^z \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\phi f(z) + \Omega_k(1+z)^2}}\right] \qquad \Omega_m + \Omega_\phi < 1 \tag{38}$$

where $f(z) = \rho_\phi(a)/\rho_\phi(a_o)$, and Ω_m, Ω_ϕ and Ω_k are the fractions of the critical energy density ($\rho_c = \frac{3}{8\pi G}H^2(a_o)$) represented *today* by the matter, scalar field and curvature term respectively (with $\Omega_m + \Omega_\phi + \Omega_k = 1$ by definition). Note that the Hubble expansion rate appears simply as a constant factor in front of this expression and therefore the knowledge of it is irrelevant when one wants to infer information about the z dependence of the integrand.

The first analyses of the supernovae data [1, 2] considered only the possibility of a cosmological constant (i.e. $f(z) = 1$, $\Omega_\phi \equiv \Omega_\Lambda$) and placed constraints in the two parameter space $(\Omega_m, \Omega_\Lambda)$, with the very surprising result that the region $\Omega_\Lambda = 0$ was inconsistent the data. In terms of the standard deceleration parameter given by

$$q_o = -\frac{\ddot{a}a}{a^2}(a = a_o) = \frac{\Omega_m}{2} - \Omega_\Lambda \tag{39}$$

a negative value is favoured, with a best fit value of $q_o = -0.55$, corresponding to a Universe in acceleration today. Admitting constraints on Ω_m inferred from other observations, one can strengthen better constrain the cosmological constant type component, with a best value falling around $\Omega_\Lambda = 0.7$.

For the more general case we are interested in how the data constrains the function $f(z)$. There is no unique way of describing such constraints and various methods have been applied to the data, or proposed in the literature for future observations ([7]). One evident possibility, given that we are working at relatively low redshift, is to try to infer information about $f(z)$ in a derivative expansion. With the present supernovae data it does not make much sense to try to go beyond first order, taking $f'(0) \neq 0$, but constant. This corresponds to taking an unknown, but constant, scaling for the new component i.e. in the notation used above, $f(z) = (1+z)^{6\xi_o}$ (thus $f'(0) = 6\xi_o$) i.e. modelling the non-standard component as a fluid with fixed equation of state $p = w_o\rho$ with $w_o = 2\xi_o - 1$. Imposing the a priori constraint of a flat Universe in the space (Ω_ϕ, w_o) the current data imply roughly [5, 6] $0.8 > \Omega_\phi > 0.3$ and $w_o < -0.6$ ($\xi_o < 0.2$) corresponding to negative values of the deceleration parameter

$$q_o = -\frac{\ddot{a}a}{a^2}(a = a_o) = \frac{\Omega_m}{2} + [3\xi_o - 1]\Omega_\phi. \tag{40}$$

Other "classical" tests have been proposed which can in principle test directly the cosmology we live in at low red-shift, but their success until now has been extremely limited for one reason or another. Number counts of objects were for a long time considered a promising probe: the idea is that at large scales the number density of a given type of object should be homogeneous, with respect to a volume which depends on the cosmology, and thus as a function of red-shift (or also of apparent magnitude) their number should vary in a way which depends on cosmology. The problem which presents itself is that the evolution of the objects typically becomes important at the red-shifts at which one becomes sensitive to cosmology. More recently observations of gravitational lensing have been explored as a possible tool [8] for constraining the equation of state of the energy in the Universe. Such lensing is sensitive to the distance travelled along the line of sight between the lensed source and the observer, and the idea is that one can infer information about this distance by trying to determine how the number of lenses changes with red-shift. Here again therefore one is essentially trying to glean information from a number count of objects as a function of red-shift, with problems inevitably arising due to uncertainties about the possible evolution of this quantity.

Beyond $z \sim 1$ there is essentially no constraint of this type on the evolution of the scale factor. Quasars, for example, of which there are now huge samples in the red-shift range $z \sim (1-5)$ have such a broad range of luminosities that we cannot infer any useful direct information from them constraining the background cosmology.

4.2 After nucleosynthesis to $z < 1$

Nucleosynthesis, as discussed in a little more detail below, allows one to infer that most of the energy density in the Universe at that redshift was in ordinary radiation. Without going beyond homogeneous cosmology to consider the constraints imposed by the observations of large scale structure (CMB fluctuations, galaxy and cluster distributions etc.) there is one further simple constraint on the behaviour of a possible scalar field energy. This is that given by the fact that the Universe both today and at nucleosynthesis is not completely dominated by scalar field. From the observations like those discussed above we have, approximately, that today $\Omega_\phi \equiv \rho_\phi/\rho_c \leq 2$. On the other hand we have also, from the temperature of the CMB, that $\Omega_{rad} \approx 2.3 \times 10^{-5} h^{-2} \sim 5 \times 10^{-5}$, where h is the Hubble constant in units of 100km/s/Mpc. Therefore we have that $\rho_\phi/\rho_{rad} \leq 4 \times 10^4$ today (with $h \approx 0.7$ [9]) so that, using the general form for the evolution of the energy density in the scalar field Eq.(15), we have a constraint

$$\begin{aligned} \frac{\rho_\phi(a_o)}{\rho_{rad}(a_o)} &= \frac{\rho_\phi(a_o)}{\rho_\phi(a_N)}.\frac{\rho_\phi(a_N)}{\rho_{rad}(a_N)}.\frac{\rho_{rad}(a_N)}{\rho_{rad}(a_o)} \\ &\approx \left(\exp - \int_{a_N}^{a_o} 6\xi(a)\frac{da}{a}\right).\Omega_\phi^N.\left(\frac{a_o}{a_N}\right)^4 < 4 \times 10^4 \end{aligned} \tag{41}$$

where the subscript N refers to nucleosynthesis, with a_N being the scale factor and Ω_ϕ^N the fraction of the energy density in the scalar field at that time, which as we will discuss in the next section is necessarily small so that the approximation $\Omega_\phi^N \approx \rho_\phi(a_N)/\rho_{rad}(a_N)$ is valid. From this we can infer the approximate bound on the evolution of the scalar field in this epoch in the form

$$\int_{a_N}^{a_o} \xi(a)\frac{da}{a} > \frac{2}{3}\ln\left(\frac{a_o}{a_N}\right) + \frac{1}{6}\ln\Omega_\phi^N - 1.8 \tag{42}$$

Since the second term on the right hand side is negative (see below), the constraint (42) is of course trivially satisfied if the scalar field energy has $6\xi \geq 4$ i.e. if it always scales at least as fast as radiation.

The bound Eq. (42) represents a non-trivial constraint only on an evolution with a period in which the scalar field scales slower than radiation. The scalar field can become in principal much more dominant over the radiation than permitted by the limit on it today; the constraint (42) simple means that that if it does so it must subsequently evolve in a mode allowing it to red-shift away sufficiently rapidly to satisfy this bound today. Consider, to take an extreme case, how long a period of inflation could intervene between nucleosynthesis and today. To calculate this we suppose

$$
\begin{aligned}
\xi &\approx 0 \qquad a < a_e \\
&\approx 1 \quad a_e < a < a_o
\end{aligned}
\tag{43}
$$

which could be implemented by a scalar field rolling in an almost exactly flat 'step' potential, with the first step at an energy scale around the nucleosynthesis scale, and a lower one at a scale like the energy density today. Fixing the initial potential by allowing Ω_ϕ^N its maximum value (~ 0.1 at 0.1MeV, see below) the constraint (42) gives $a_o/a_e > 1.5 \times 10^5$. Thus a period of inflation could follow shortly after nucleosynthesis (with growth of scale factor by $\sim 10^3$ i.e. about 7 e-foldings) lasting until a little before matter-radiation equality, and then followed abruptly by a period of 'kination' lasting until the present epoch (at which point it could become inflationary again).

4.3 Structure Formation

The previous example is an extreme one given to highlight how little observations which probe the homogeneous Universe actually tell us about the cosmological history between nucleosynthesis and today. Once the possibility is admitted of an important role for coherent scalar field energy today in order to explain observations, it is however quite natural to envisage such possibilities. While it is easy to rule out a model as "wild" as that just given from considerations of structure formation, as we will now discuss, one should not be so hasty in throwing out some unconventional, albeit less extreme possibilities of this type. In fact one model we will discuss in section 7 uses the idea that a recurrent role for scalar fields in the cosmological history is a possible way of resolving the so called "coincidence problem" associated with the acceleration of the Universe today.

How do observations related to structure formation constrain the energy content of the Universe as a function of red-shift? Such observations are essentially of two sorts: measurements of the fluctuations in the CMBR and of the distributions in space (or angle) of discrete objects (galaxies/clusters/quasars etc.). The constraints one places on a scalar field contribution to the energy density are of course in this case very model dependent. In particular they will depend on what kind of assumptions one makes about initial conditions for perturbations. In the case of the distributions of discrete objects, in particular at the smaller scales at which non-linear effects become essential in understanding the formation of structure, the kind of conclusions one reaches will depend in particular on the details of how the visible matter is understood to be related to that of dark matter. Studies taking a wide range of such data into account have been used to constrain cosmologies with very specific and limited assumptions about the scalar field component [10].

Rather than attempt any exhaustive discussion of this question we restrict ourselves to a few basic observations about the effect of adding energy in a scalar field component during the era of structure formation (by which we mean, in practice, back to the red-shift when the horizon scale is considerably smaller than that at decoupling i.e. $z \sim 10^5 - 10^6$). The first observation concerns the effect a change of equation of state has on the size of physical scales today. The physical size today of the causal horizon at time t is

$$
R_H(t) = a_o \int_o^t \frac{dt}{a(t)} \equiv a_o \eta(t) \tag{44}
$$

where $\eta(t)$ is the conformal time. Since $a \propto t^{1/3\xi}$ we have, for $\xi > 1/3$ (i.e. subluminal expansion)

$$
a \propto \eta^{\frac{1}{3\xi - 1}}. \tag{45}
$$

Therefore the ratio of the scale today corresponding to the horizon scale at the time of decoupling to the horizon today is, assuming ξ fixed since decoupling,

$$\frac{\eta_{dec}}{\eta_o} = \left(\frac{a_{dec}}{a_o}\right)^{3\xi-1} \approx \left(\frac{1}{z_{dec}}\right)^{3\xi-1}. \tag{46}$$

For a flat cosmology this is essentially just the angular size (in radians) subtended by the horizon scale at decoupling on the CMBR sky observed today. Taking, as in the example of the previous example, $\xi \approx 1$ instead of the usual matter scaling value $\xi = 0.5$, we find that this angle is smaller by a factor of $\sim 10^{-3}$ than the canonical 2^o. Therefore any standard type model of structure formation will immediately rule out such a model, from the observations [11] of the structure in the spectrum of CMBR fluctuations. Indeed given the precision in recent measurements on the location of the first peak, one can clearly constrain the equation of state to be very close to that in the standard matter/radiation cosmology (to an accuracy which depends on what prior assumptions one makes restricting the initial perturbations and other cosmological parameters). One could still derive, however, some quite particular cosmological model like our "wild" one above, allowing a combination of an inflationary epoch and a kination epoch after decoupling, just so as to have this first peak at the right location. Such a possibility is ruled out by the second strong qualitative constraint we mention: the growth of structure, in particular at smaller scales than those probed (at least currently) by the CMBR measurements, requires that matter must dominate the energy density of the Universe for a reasonably long time. For, in a standard perturbation analysis (in k space) of the growth of fluctuations under gravity, incorporating a scalar field like we that we have considered as well its fluctuations ψ, one finds (see [33]) for the equations governing the growth of the fluctuations δ_m in the CDM

$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m - \frac{3}{2}\mathcal{H}^2(\Omega_m\delta_m + 2\Omega_r\delta_r) - 2\dot{\phi}\dot{\psi} + a^2V'(\phi)\psi = 0 \tag{47}$$

where here derivatives are with respect to conformal time, with $\mathcal{H}$ the correspondingly defined Hubble expansion rate. The perturbations in the scalar field are generically damped themselves, and the associated terms in ψ can be neglected. The growth of structure is driven by the term proportional to Ω_m, and a value not too far from the canonical one, in which $\Omega_m \approx 1$ for most of the time after decoupling, is required. Adding a dominant component of scalar field for any period will thus essentially freeze structure formation during this time. If such dominance last for too long, there will be no time for structure to form.

These constraints of course still leave space for contributions from scalar field - we have not quantified exactly how much. The current popularity of ΛCDM and quintessence models are evidence that such a contribution is not only tolerable, but consistent with constraints imposed from observations. We will discuss below some models involving a small contribution from a scalar field, but acting right through the era of structure formation, which are quite compatible with structure formation, indeed producing signatures in the CMBR which should be observable in future experiments.

4.4 Nucleosynthesis

The only other direct measure of any precision constraining directly the content of the right hand side of the Einstein equation during cosmological history (and indeed it is of a precision much greater, at least for the moment, than given by those constraints just discussed) is that coming from nucleosynthesis. Indeed it is one of the great successes of the standard Big Bang model that when extrapolated back to nucleosynthesis, when the Universe was hotter by a factor of 10^{10}, it gives predictions for abundances of the light elements which are in very good agreement with observations. The predictions from nucleosynthesis depend directly on how the expansion rate of the Universe varies as a function of temperature at that epoch ($T \sim 1 - 0.05$MeV). If we take the matter content of the Universe to be that of the standard model of Particle Physics, the expansion rate is completely specified and the relic abundances are predicted as a function of the baryon-to-entropy ratio. For very reasonable values of the latter (i.e. reasonable given the densities of baryonic matter we observe today and the temperature of the CMBR) good agreement is found with the relative abundances of Helium, Deuterium and Lithium. The standard way in which deviations from this expansion rate have been studied is in terms of the addition of an unspecified number of additional relativistic degrees of freedom ΔN_{eff} (effective fermionic degrees of freedom by convention because of the interest in constraining the number of neutrino species). Since this extra energy scales like radiation (by supposition) the bounds on ΔN_{eff} can be converted, strictly speaking, to a bound on a fraction of energy in a scalar field scaling as $1/a^4$ through the simple relation

$$\Omega_\phi^{nucl} = \frac{7\Delta N_{eff}/4}{10.75 + 7\Delta N_{eff}/4}, \tag{48}$$

where 10.75 is number of effective degrees of freedom in the standard model of particle physics (with three massless neutrinos, or variants with three massive neutrinos sufficiently light that they are relativistic at ~ 0.1 MeV). What precisely this bound gives depends on precisely what nucleosynthesis data one takes, a subject on which there has been considerable debate in the last few years [12]. The most conservative bound in the literature of the last few years is $\Delta N_{eff} < 1.5$ [13], corresponding to $\Omega_\phi < 0.2$, while the strictest bounds are $\Delta N_{eff} < 0.2 - 0.3$, values closer to which have been favoured by more recent analyses [14], corresponding to $\Omega_\phi < 0.03 - 0.045$. In the rest of this paper, unless otherwise states, we will take the bound $\Omega_\phi < 0.045$ advocated in a recent analysis [15] as our fiducial value.

As noted this is calculated directly as a bound only for the case of a scalar field component scaling like radiation. Without going into a full analysis of nucleosynthesis for the completely general case we are interested in - a calculation which to the our knowledge has never been performed - one can infer approximately what the bound is in this case. The change to the expansion rate given by adding a radiation like energy feeds at an important (i.e. observable) level into the final abundances only for the Helium. The primary effect is through the modification to the temperature of freeze-out of the weak interactions, which fixes the crucial ratio of neutrons to protons at this time, while there is also an effect on the time at which the Helium is synthesized ($T \approx 0.06$ MeV). When the expansion rate is increased both effects go in the same direction - towards a higher Helium abundance. One can infer that a component scaling away faster than radiation (i.e. becoming relatively less important in time) will certainly be consistent with observations if the same bound $\Omega_\phi^{nucl} < 0.045$ is satisfied at freeze-out $T \sim 1$ MeV; indeed a slight weakening of the bound will occur due to the fact that the synthesis time will be closer to that in the standard case. For a component scaling slower than radiation the bound at 1 MeV will become stricter as the duration of nucleosynthesis is further shortened. In the extreme case of a cosmological constant we can be sure that the same bound as for radiation scaling, but applied at the temperature $T = 0.06$MeV (corresponding to a bound $1/(0.06)^4$ smaller at $T = 1$ MeV) will be safe, but a full analysis of this case would be needed to determine the exact bound. In the calculation in the previous section we assumed for this case the bound $\Omega_\phi(0.1\text{MeV}) < 0.1$.

One recent analysis [16] of the effect on nucleosynthesis of more radically changing the Einstein equation considers the case of an expansion rate $H = H_I(T/1\text{MeV})^\alpha$, and treats H_I and α as free parameters. This models the case that the scalar field with a fixed scaling law completely dominates over the radiation right through nucleosynthesis (with $0 < \alpha = 3\xi < 3$ since $a \propto t^{1/3\xi}$). While He or De constraints taken singly admit a class of radically different solutions (with expansion rates several orders of magnitude larger giving the same abundances), the combined constraints on He, De and Li are shown to give a very limited, almost one dimensional allowed region in the parameter space of H_I and α, centred on the known standard solution. There are non standard solutions with H_I larger than in the standard case in the range $2 < \alpha < 3$, but H_I increases in this range by less than an order of magnitude. Since the expansion rate is normalized at 1MeV it follows that these cases cannot describe the case of co-existence of energy in a scalar field and the standard radiation component, since with these values of H_I the radiation would return to domination before the end of nucleosynthesis. It thus seems that there are no non-standard solutions which would circumvent the bounds discussed in the previous paragraph, although a direct analysis of this case in its full parameter space would be required to prove this definitively. Likewise one could study what the joint bounds are when chemical potentials are allowed vary and whether new solutions exist. These are rather ptolemaic possibilities which would require more direct motivation to be taken seriously.

4.5 Before nucleosynthesis

Prior to nucleosynthesis we have *no* direct probe of cosmology. We only know that it must be such as to produce a universe like the one required at nucleosynthesis and after, on which we do have constraints such as those just discussed. The Big Bang cosmology, because of the intrinsic problems it presents when extrapolated back arbitrarily in time (singularity at the origin of time, horizon problem, flatness problem etc.) demands the construction of a larger framework from which it emerges. The most popular such construction for the last two decades has been given by inflationary models, which are essentially (at least in most cases) scalar field dominated models like those we have discussed with $\xi \to 0$. The inflationary phase matches on to the FRW phase in a two stage process: first, the scalar field enters a region of its potential in which it can oscillate and in which (typically) there is an equipartition of kinetic and potential energy so that $\xi = 1/2$ and $\rho_\phi \propto 1/a^3$; subsequently this energy decays into particle-like degrees of freedom which eventually become the radiation driving the FRW phase. From this time (characterized approximately by the "reheat temperature" T_{rh}, typically somewhere a little below the GUT scale) the Universe is supposed to be in a radiation dominated phase, with an expansion rate simply given by the appropriate number of relativistic degrees of freedom.

In principle there is no reason why prior to nucleosynthesis scalar fields could not dominate in a non-inflationary modes, in particular with a phase of what we termed kination. This is one of the models we will discuss at length below, describing some of the (potentially observational) effects it can have on cosmology. These signatures are of an indirect type, analagous to those of nucleosynthesis. One of the generic features of such a phase, which is in principle directly observable, is that it leads to the production of gravitational waves. While again this does not put any significant constraint currently on the duration of such a phase, in the future it is conceivable [17] that such a signature may be found, in particular in the context of the kind of alterative reheating model we will discuss in section 8.

5 Three modifications of FRW cosmology

If we put no apriori constraints on the potentials allowed for scalar fields, the only constraints we have on possible modifications of the standard FRW model without such a component are given by the observational constraints discussed in section 4. This leaves open in principle a vast range of possibilities. We will concentrate in the rest of this paper on three possible such modifications, which we now introduce briefly. Standard (primodial) inflation is, as we have mentioned, the best known case which we choose explicitly not to discuss in this paper.

5.1 Kination before nucleosynthesis

The observation that the energy in a kinetic energy dominated mode of a rolling scalar field scales away faster than in radiation suggests a simple possibility: a Universe dominated prior to nucleosynthesis by such a mode. We refer to such a phase as *kination*. We will thus suppose that the expansion rate in the early Universe is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_e}{2}\left[\left(\frac{a_e}{a}\right)^m + f(a)\left(\frac{a_e}{a}\right)^4\right], \tag{49}$$

with $m > 4$. We will refer to the particular case of $m = 6$ ($\xi = 1$) as *pure kination*. The transition from the phase of kination to radiation domination comes about of itself as the first term red-shifts away to subdominance, with a_e corresponding to the scale factor at this transition from kination to radiation domination. The factor $f(a)$ accounts for the effect of decouplings, which, if they are assumed to occur adiabatically, is given by $f(a) = [g(a_e)/g(a)]^{1/3}$, where $g(a)$ is the number of relativistic degrees of freedom as a function of the scale factor a.

We have noted in section 2 that, in terms of our notation for scalar fields, $m = 6\xi$ which leads to the expansion law $a \propto t^{2/m}$ during kination, and therefore, given that $a \propto T^{-1}$, we find

$$H_{kin} \propto T^{\frac{m}{2}} \qquad 4 < m \leq 6 \tag{50}$$

so that the expansion rate begins to increase faster as a function of temperature than in the standard radiation dominated cosmology as we extrapolate back to temperatures prior to nucleosynthesis. The nucleosynthesis constraints we have discussed previously can be stated as a lower bound on T_e, the temperature at the time of equality of radiation-scalar field energy density. If Ω_ϕ^N is the greatest allowed fraction of the energy in the scalar field at 1MeV, then the bound on T_e

$$T_e \geq \left(\frac{1}{\Omega_\phi^N}\right)^{\frac{1}{m-4}} \text{MeV} \tag{51}$$

or $T_e \geq 4.5$ MeV for $\Omega_\phi^N = 0.05$ and $m = 6$. Alternatively at any temperature well above the nucleosynthesis scale to which we extrapolate the phase of kination we have an upper bound on the expansion rate given by

$$H_{kin}(T) \leq H_{rad}(T)\left(\frac{T}{T_e}\right)^{\frac{m-4}{2}} \tag{52}$$

where $H_{rad}(T)$ is the standard radiation dominated expansion rate extrapolated back to temperature T. (The effect of decouplings have been neglected here). For the limiting case of $n = 6$ the ratio of the expansion rate to the standard one increases in linear proportion to the temperature. As we go back to epochs much before nucleosynthesis this can have enormously important effects on any calculation of effects coming from cosmology at this time. We will briefly discuss some examples below.

From the discussion in the previous section we can immediately see what kind of simple potential could lead to an implementation of this scenario. The most trivial way is with an exactly (or sufficently) flat direction of a potential in which the energy scales as $1/a^6$. What is required is the dominance of the energy density in this mode over that in the radiation, a condition whose fulfillment we will discuss further below. Alternatively, any scaling faster than that of radiation can be obtained with the simple exponential potential of Eq. (27) with $\lambda > 2$, when the scalar field energy dominates over the radiation. Given the nature of the attractors we have discussed which exist in this case, it is clear that we can satisfy the constraint that the scalar field energy be sub-dominant at nucleosynthesis if λ is sufficiently large so that $\Omega_\phi = 4/\lambda^2 \leq 0.045$ (taking the more recent stricter bound described above) i.e. $\lambda \geq 10$. In this case however we have necessarily $1/a^6$ scaling in the kination phase. To implement a phase in which the scalar component scales, say, as $1/a^5$, which corresponds to $\lambda = \sqrt{5}$ for the simple exponential will require a modification of the potential. We will return to this kind of consideration in the next section when we discuss the question of fine-tuning in these models.

We briefly consider now some examples of the implications of such a modification of the standard cosmology. The reader less interested in early Universe cosmology may wish to proceed to section 5.2.

5.1.1 Baryogenesis at the EW scale

One of the few possible probes of cosmology prior to nucleosynthesis is through electroweak physics which describes the Universe when extrapolated back to temperatures $T \sim 100$GeV. One attempt to link the knowledge attained at accelerators of the physics at this scale to cosmology at this scale is electroweak baryogenesis[25]. In analogy to how nucleosynthesis explains the trace abundance of light elements, one tries to see if electroweak physics can explain the origin of the one crucial input to nucleosynthesis, the ratio of the number of baryons to photons (stated usually as the baryon to entropy ratio $n_B/s \sim 10^{-10}$). Just as in nucleosynthesis the cosmology one assumes, which determines the expansion rate as a function of temperature, is important in the calculation. Usually in this context the simplest extrapolation of cosmology is assumed from nucleosynthesis i.e. the standard FRW model dominated by radiation with the number of degrees of freedom given by the standard model. In this context one finds that while electroweak models have all the elements necessary to create a baryon asymmetry, satisfying the three conditions of Sakharov (violation of B and CP, departure from thermal equilibrium) in practice it is extremely difficult to create a baryon asymmetry of the required order. The minimal standard model(MSM) presents the particular problem that its violation of CP is too suppressed. While this problem is not one in many typical extensions of the MSM (e.g. multi Higgs models, the minimal supersymmetric model), all models face the so called "sphaleron" bound, which reduces the viable parameter space to a very small region in most cases of interest.

This bound comes simply from the requirement that the baryon violating processes in the electroweak model be inoperative after the electroweak phase transition, and therefore involves a comparison of the "sphaleron" rate (of baryon violating processes) with the rate of expansion of the Universe at that time. In [4, 26] we have studied how this bound depends very generically on the expansion rate at the electroweak scale. It is usually stated (with the implicit assumption of the standard radiation dominated expansion rate) as an absolute bound on the ratio ϕ/T of the vacuum expectation value (vev) of the Higgs condensate to the temperature, at the transition temperature. In Figure 1 is shown the minimal required value of ϕ/T as a function of the expansion rate (treated as an undetermined variable). This bound on ϕ/T must subsequently be translated into a constraint on the zero temperature observable parameters in any particular model. While the results already show a substantial dependence of the bound in this form on such parameters, and on the details of the phase transition through the temperature at which it occurs, one sees that in all cases over the range explored the bound with a modified expansion rate can be written to a good approximation as

$$\frac{\phi}{T} > \frac{\phi}{T}|_{rad} - (0.085) \ln \left(\frac{H}{H_{rad}} \right) \tag{53}$$

where $\frac{\phi}{T}|_{rad}$ is the usual bound, when the expansion rate is H_{rad} at the phase transition. The simple logarithmic dependence comes from the fact that the rate of baryon number violation is primarily dependent on the vev through an exponential. How this translates into a bound on parameters is shown for the case of the MSM in Figure 2, which shows the minimum expansion rate required as a function of the Higgs mass. Given that the greatest modification to the expansion rate in the context we are discussing will be

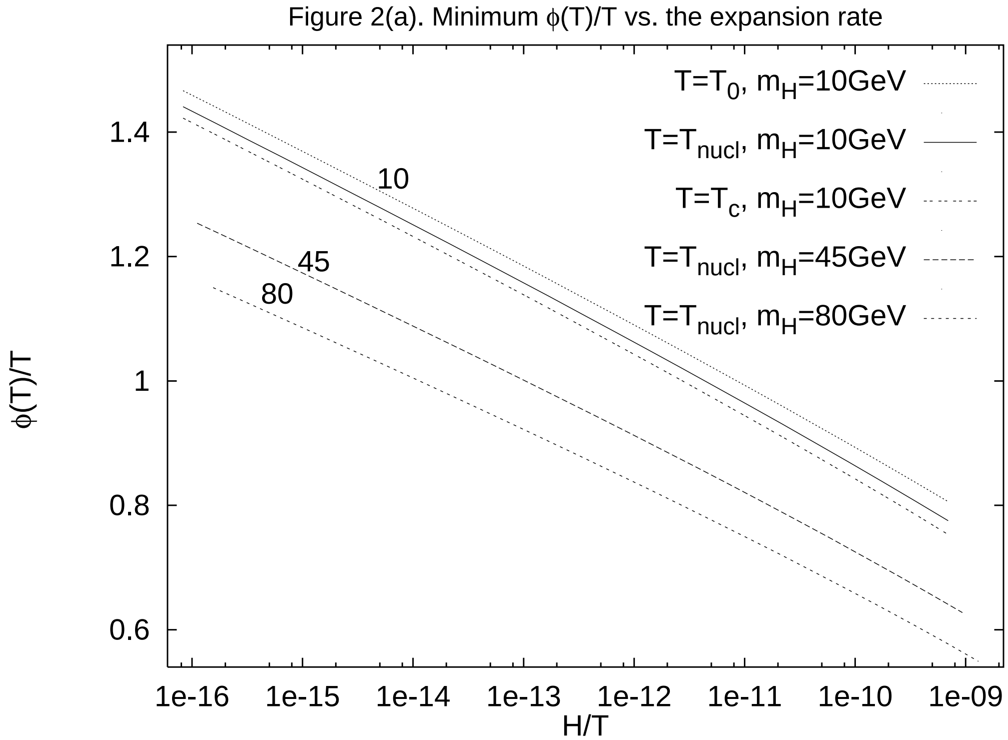

Figure 1: The minimum value of ϕ/T required in the broken phase during the EW phase transition to avoid washout of baryon asymmetry, as a function of the expansion rate of the Universe. The calculation is for the standard model, and one sees some dependence on the Higgs mass, and the temperature at which the transition takes place (T_c is the critical temperature where the broken and unbroken phases have the same free energy, T_{nucl} is the (slightly lower) temperature at which the broken phase bubbles nucleate, and T_o is the temperature below which $\phi = 0$ is no longer a local minimum).

given in the case of pure kination ($m = 6$) so that

$$\frac{H}{T} \leq 1.8 \times 10^{-11} \left(\frac{T_c}{100\text{GeV}} \right)^2 \tag{54}$$

it is clear that the sphaleron bound in the MSM will never be satisfied in this cosmology either. In fact the final LEP bound on the MSM Higgs mass excludes all of this parameter space anyway. In extensions of the MSM, which as we have mentioned must be considered anyway because of the need for CP violation beyond that in the MSM, there are small regions in the experimentally allowed parameter space still compatible EW baryogenesis, in the canonical radiation dominated case [27]. With the modification allowed by Eq.(54) the bound (53) gives $\frac{\phi}{T} > \frac{\phi}{T}|_{rad} - 0.6$ which widens very considerably the compatible parameter space.

The sphaleron bound refers to the case that there is a phase transition of first order at the electroweak scale. This is usually the only possibility believed to be viable, as otherwise the only non-equilibrium effect (which is essential to the creation of the baryon asymmetry) comes from the expansion of the Universe. A very simple argument gives that the time dependence induced through the expansion of the Universe can give rise to a baryon asymmetry

$$\frac{n_B}{s} \sim \frac{\delta_{CP}}{g_*} \frac{H}{T} \tag{55}$$

where δ_{CP} is some dimensionless quantity characterising the strength of CP violation (naively expected to be of order one, at most) and g_* (~ 100) is the number of relativistic degrees of freedom at the electroweak

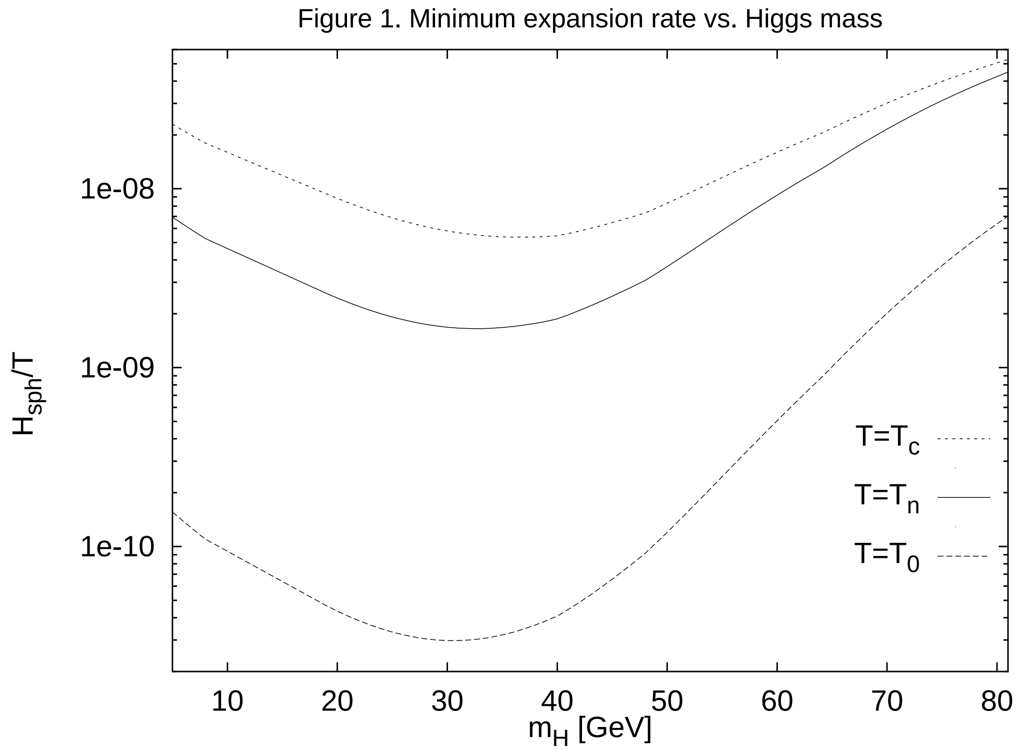

Figure 2: The minimum expansion rate required to satisfy the sphaleron bound in the minimal standard model, as a function of the Higgs mass m_H. The different temperatures are as in Figure 1.

scale. While in the standard cosmology with $H/T \sim 10^{-16}$ baryogenesis at the required level is completely unfeasible, when we take the bound given by Eq. (53) this is not so evidently the case. A more detailed investigation is motivated by the fact that this case (where the transition in the electroweak model is not first order) acually includes the large part of the experimentally allowed parameter space in such models. Such an investigation shows [26] that if we go close to saturating the upper bound in Eq. (54) it may indeed be possible to create a baryon asymmetry of the required order in this case. In particular the naive estimate of Eq. (55) misses out on the fact that the time variation of the vev around the cross-over temperature can be quite enhanced compared to the expansion rate, roughly cancelling the factor g_*, Further the appropriate T_c in (53) typically lies well above 100GeV in this case [25]. The CP violating factor can easily be of order unity consistently with zero temperature laboratory constraints, and one has $n_B/s \sim H/T$. In this scenario then the baryon asymmetry is simply a direct measure of the expansion rate of the Universe at this time. If accelerators in the future discover a symmetry breaking scalar sector beyond the MSM which corresponds to a second order phase transition or crossover at the electroweak scale, this would be one possible explanation of the baryon asymmetry in this context (provided there is also the CP violating structure required).

Such an outcome on its own would not merit the conclusion that the expansion rate at the electroweak scale was really the corresponding one. After all we have only one observable here (the ratio n_B/s) and one unknown parameter (H/T)! Note, however, that the condition that there exist *some* value of the expansion rate at the electroweak scale consistent with the production of the required baryon asymmetry is not at all a trivial one: it requires very specific features in the physics (in particular CP violation in a scalar sector) beyond that we know at present (essentially the MSM plus some neutrino mass sector). Such a discovery at accelerators would therefore provide a very strong motivation for considering very seriously the possibility that the Universe is not radiation dominated before nucleosynthesis. And in particular if the expansion rate required were consistent with a phase of kination (i.e. consistent with Eq. (54)) this particular possibility would have to be taken very seriously. On its own, however, baryogenesesis considerations cannot even in principle do more than this. Some other probe of cosmology before nucleosynthesis would be required. A good analogy is perhaps that with nucleosynthesis itself: If we were only to measure the primordial abundance of Helium its predictive power is extremely limited (given that we do not know a priori n_B/s with much precision, nor the expansion rate). It is with the measurement of Deuterium (and subsequently Lithium) abundances that it becomes a non-trivial and much more impressive framework. In this analogy the baryon asymetry is our Helium. To probe the pre-nucleosynthesis era strongly we will also need a Deuterium. The next section gives one possible candidate.

5.1.2 Dark matter freeze-out

A central element of any current standard type cosmological model is non-baryonic dark matter, or indeed, more specifically, cold non-baryonic dark matter : the Universe is supposed to contain a significant fraction of its energy density in particles uncoupled (or very weakly coupled) to ordinary matter which cluster efficiently under gravity. Particle physics readily comes up with candidates for this dark matter. The neutrino is the only known particle which is of this type, but as a candidate for cosmological dark matter it does not have the required properties (it is 'hot' instead of 'cold'). A generic scenario involves a weakly interacting massive particle (WIMP): being weakly interacting it drops out of equilibrium with ordinary baryonic matter early in the Universe at a temperature T_{dec}, but it is sufficiently massive that it is already non-relativistic at this time (i.e. $M >> T_{dec}$). It is the latter property that makes it 'cold' (i.e. it has small initial velocity dispersion) and efficient in its gravitational collapse properties at small scales. Extensions of the MSM generically produce candidates of this type with masses around the electroweak scale. Given the form of their couplings, there are then two free parameters: their mass and the amplitude of the relevant coupling. For given values of these parameters, and *assuming the expansion rate is known*, one can determine when the particle decouples, and therefore what its abundance is in the Universe. To be a candidate for cosmological dark matter this abundance must fall within a very specific window (for the currently favoured cosmology $\Omega_{CDM} \sim 0.3$) so that a one-dimensional subspace is singled out in the two dimensional parameter space of mass and coupling. It is then the task of dark matter searches (either directly designed for this purpose, or indirectly in accelerator experiments looking for physics beyond the MSM) to try to cover this space and rule out (or discover) the dark matter candidate.

As we have noted these considerations depend directly on the assumption that the expansion rate is known. Typically the decoupling in the standard radiation dominated FRW model takes place at pre-nucleosynthesis scales, when $T \sim 100$ MeV. Sensitivity to the fact that the expansion rate at this scale is actually unknown has been considered for example in [28] and [29][4]. Very qualitatively it is easy to see that the effect can be important as follows. We must compare the relevant weak interaction rate $\Gamma_w(T) = n_w \sigma v$ (where the symbols have their conventional meanings) to the expansion rate of the Universe. Since the particle is non-relativistic when it decouples the main sensitivity to the temperature is through an exponential $\sim e^{-m_w/T}$ in the abundance factor n_w (where m_w is the particle's mass). If we increase the expansion rate as a function of temperature, the effect is approximately to increase the decoupling abundance, and thus the final abundance, in linear proportion to the expansion rate, obtaining a result rather like that in Eq. (55). If such decoupling takes place around 100MeV, given the bound (52), the expansion rate may be larger by a factor of about twenty. Just as in the case of baryogenesis the possibility

[4]As alternative cosmology the former had specifically an anisotropic Universe in mind, which can be treated in certain cases in terms of an effective additional component scaling as $1/a^6$. The latter is more generic in its considerations and indeed mentions the possibility of a scalar field oscillating about its minimum.

of turning our speculation on the possibility of a FRW model with a phase of kination before nucleosynthesis into a physically testable theory (in the same sense in which nucleosynthesis is such a theory) would have to involve the discovery of a WIMP particle which could play the role of CDM in standard models of structure formation. Once the coupling and mass are determined its abundance would be predicted as a function of the expansion rate at its decoupling time. If a cosmologically interesting abundance (i.e. one consistent with this WIMP being the CDM in a well tested model of structure formation) resulted from an expansion rate corresponding to kination ending just before nucleosynthesis, such a possibility would certainly need to be taken very seriously. If the *same* expansion rate did both this and created the right baryon asymmetry, this possibility would surely become a theory to be considered with the same weight as nucleosynthesis.

5.1.3 Baryogenesis at higher energy scales

In all viable models of baryogenesis the baryon asymmetry is created at cosmological epochs well before nucleosynthesis. The reason for this is simply that the required violation of baryon number must be efficient at the time of baryogenesis, while giving rise to baryon violation so suppressed at low temperature to be consistent with the extremely tight constraints coming from experiment (principally from the non-observation of proton decay).

Besides electroweak scale baryogenesis, which makes use of the very elegant way in which the MSM violates baryon number, there are other schemes, generally involving physics at higher energy scales. The prototype is GUT baryogenesis. In this context, in which leptons and quarks are part of the same representation of the gauge group, baryon violating processes at the perturbative level appear generically, and there is no reason why CP violation should be small (it is clearly unobservable in the corresponding sector!). Baryogenesis occurs when a particle coupled to these processes drops out of equilibrium and subsequently decays. (That it drop out of equilibrium is required by the Sakharov conditions for baryogenesis). The asymmetry produced is typically then independent of the expansion rate taking a form like

$$\frac{n_B}{s} \sim \frac{10^{-2}}{g_*} h^4 \delta_{CP} \tag{56}$$

where h is the corresponding coupling, and the suppression of 10^{-2} and the power of the coupling arise from the fact that the CP violating effects arise at loop level through interference terms between diagrams. A variant on this idea is 'leptogenesis'. Here one has perturbative lepton number violating processes at a high energy scale which produce a lepton asymmetry at an out of equilibrium decay, which then subsequently leads to a baryon number through the same baryon and lepton number violating processes of the MSM used in electroweak baryogenesis. This is a model which has become particularly interesting and well motivated in the context of experimental results in recent years indicating clearly that neutrinos have mass. Indeed the simplest schemes accommodating neutrino masses make use of lepton number violating Majorana mass terms, and a "see-saw" mechanism associated with a high mass scale which becomes that associated with the out of equilibrium decay. In the standard radiation dominated cosmology this is a temperature around 10^{10}GeV.

The effect of changing the pre-nuclesynthesis expansion rate as envisaged in a phase of kination might at first appear not to be so interesting: The baryon asymmetry does not directly depend on the expansion rate, but only indirectly through the out of equilibrium constraints. One would thus expect that changing the expansion rate will simply modify the range of masses and couplings which give the right baryon asymmetry. Unlike in the previous cases there is no prospect of measuring these parameters directly, so this would seem to be of no more than theoretical interest.

In fact the change brought to these schemes is of a different nature, and of zero order qualititative importance. In any model of baryogenesis one must not just create a baryon asymmetry; one must also be sure that this baryon asymmetry doesn't dissappear again before nucleosynthesis. What certainly can make it disappear are the baryon number violating processes of the MSM. As we have discussed in the context of electroweak baryogenesis above, these processes switch off around the electroweak phase transition at temperatures $T \sim 100$GeV. The "sphaleron bound" we discussed is simply the condition that this freeze-out occur at the transition itself, so that the baryons created then can survive. Even if it is not satisfied these processes always switch off shortly after the transition (because the rate depends through an exponential on the temperature). For any mechanism of baryogenesis we must check that the baryon asymmetry at this "freeze-out" temperature T_{freeze} is that required at nucleosynthesis. To calculate this asymmetry at any time we can use a simple thermal equilibrium calculation

$$\langle B \rangle = Tr\left(Be^{-(H-\mu_A Q_A)/T}\right) \tag{57}$$

where μ_A are the chemical potentials associated to the conserved charges Q_A. Using the CPT invariance of the Hamiltonian H, and the invariance of the thermal trace under T, one can show easily that $\langle B \rangle \neq 0$ only if $\langle Q_A \rangle \neq 0$ for *some* CP odd charge. What these charges are depends on the particle physics which is operative at the electroweak scale, which is of course tightly constrained by accelerator experiments. In the MSM there are only three exactly conserved charges[5] $\frac{1}{3}B - L_i$, where L_i are the individual lepton numbers of the three fermionic generations(L_e, L_μ, L_τ). If all processes are assumed to in equilibrium it follows that the baryon asymmetry will be zero unless one of these global charges was violated at some point during the evolution of the Universe. In simple GUTS, like $SU(5)$, these charges are in fact all conserved, which leads to the conclusion that, whatever the baryon asymmetry created is, it will be washed out subsequently at temperatures at which the equilibrium calculation given is valid.

Another conclusion one draws from this analysis is that all one needs to do to create a final baryon asymmetry is to violate one of these conserved charges of the MSM i.e. one does not need to violate specifically baryon number B. If we manage to create such a charge through the violation of this charge at some scale, it will subsequently give rise through the B-violation of the MSM to a baryon asymmetry. Leptogenesis is an example of a model which exploits this: it involves additional particle content in neutrinos giving rise to L violation. In order to produce through a "see-saw" the tiny masses now believed to be those of neutrinos, one allows lepton number violating Majorana mass terms in the ordinary MSM neutrino as well as in an additonal heavy neutrino degree of freedom. To produce the observed masses the heavy mass scale is typically of order 10^{10}GeV. When the associated particles decay they violate lepton number and can have CP violating structure in their mass matrices which leads to a lepton asymmetry in the light leptons. One must of course be careful to ensure again that this asymmetry survives i.e. that there are at all times conserved CP odd charges to "protect" it. In particular one needs to be sure that *all* lepton number violating processes are out of equilibrium at the electoweak scale.

These conclusions about what is necessary for the creation of a final baryon asymmetry were predicated on the assumed validity of the equilibrium calculation, with the conserved charges being the exactly conserved ones of the MSM. We thus assumed implicitly that all processes (perturbative and non-perturbative) of the standard model are in equilibrium around the electroweak scale. For this to be true the rates of all such processes must be faster than the expansion rate at that time. In the standard cosmology this is true, but it may change as the pre-nucleosynthesis cosmology does. The slowest perturbative process in the MSM is that mediated by the electron yukawa coupling y_e, which is the smallest coupling ($y_e \sim 10^{-5}$) simply because the electron is the lightest fermion. In the standard radiation dominated cosmology the

[5] These are the conserved global charges. We assume the gauge charges are zero.

corresponding processes, which are the only ones violating right-handed electron number e_R, come into equilibrium at a temperature T_o when

$$y_e^2 T_o \sim \sqrt{g_*}\frac{T_o^2}{M_P} \tag{58}$$

which a more careful calculation shows to correspond to $T_o \sim 10$TeV [30]. With the modified expansion rate in a pure kination phase (with $m = 6$) ending at temperature T_e this temperature is modified to $T \approx \sqrt{T_o T_e}$ which can be as low as $T \sim 7$GeV i.e. well below the electroweak scale. In fact we only need $T_e \leq 1$GeV in order for the right handed electrons to be out of equilibrium at the electroweak phase transition, when the baryon number violating processes ("sphaleron" processes) of the MSM switch off abruptly.

This leads to an important conclusion. In the standard radiation dominated cosmology one had the result that any non electroweak model which gives rise to baryogenesis would have to violate a conserved charge of the standard model. In our modified cosmology this is no longer true. An example [31] is a model which can produce a right-handed electron asymmetry (what we termed *electrogenesis* in [31]). In the presence of such an asymmetry, the baryon number violating processes of the standard model, when they are in equilibrium above the electroweak scale, lead to a baryon asymmetry, which will survive until when these processes switch off (and thus until nucleosynthesis) if the right-handed electrons remain out of equilibrium until below the elecroweak scale. In a simple GUT like minimal $SU(5)$, which has the *same* global symmetries as the MSM, while no final baryon asymmetry can be produced in the standard cosmology, the modification by a phase of kination beginning before the GeV scale makes the production of the baryon asymmetry feasible. One must simply produce an asymmetry in right-handed electrons $e_R/s \sim 10^{-10}$, which one can envisage happening in a standard out of equilibrium decay: the right handed electrons appear in the 10 of SU(5) and their number is violated by the gauge interactions above the GUT scale. As the superheavy bosons decay out of equilibrium they will produce a non-zero value of e_R (if there is CP violation) given roughly by an expression like Eq. (56).

In developing detailed models of this type it is important to bear the following point in mind.

5.1.4 A limiting temperature

We have here so far naively supposed a backward extrapolation of the phase of kination to some arbitrarily large temperature. A closer examination shows that quite some caution needs to be used in this case.

The first limit one might apply to this extrapolation is given when the energy density reaches the Planck density. While in the usual radiation dominated cosmology just gives $T_{lim} \sim M_P$, in pure kination ($m = 6$) it leads to

$$T_{lim} \sim M_P \left(\frac{T_e}{M_P}\right)^{1/3} \tag{59}$$

where T_e is the temperature at which the radiation dominated epoch begins. If the latter is close to its lower limit, just above the nucleosynthesis scale at $T_e \sim 5$MeV, this limit can be as low as $T_{lim} \sim 10^{12}$GeV.

If we look further we find that already before this the naive backward extrapolation becomes invalid. Let us consider the meaning of the use of temperature. In the Einstein equation the temperature refers to that of the (highly relativistic) particle-like degrees of freedom which are in equilibrium. In any typical particle theory the couplings of the most rapid processes will be of order unity, so that the characteristic interaction rate of a particle at high temperature will be $\Gamma \sim T$. For it to be consistent to use the temperature we require $\Gamma \geq H$. In the standard cosmology this means we need $T \geq \sqrt{g_*}T^2/M_P$, or $T \leq g_*^{\frac{1}{4}} M_P$, above the previous upper bound T_{lim} in this case. For our modified cosmology, things are quite different. Taking the limiting case of $n = 6$, the condition for equilibrium of the fast processes becomes much stronger:

$$T_{eq} \leq g_*^{\frac{1}{4}} \sqrt{T_e M_P} \tag{60}$$

which for the lower limit $T_e \sim 5\text{MeV}$ gives $T_{eq} \leq 10^8 GeV$.

Formally we can apparently extrapolate back beyond this scale. The physical meaning of the temperature would then just as a parametrization of the mean energy density in the radiation which will continue to scale as $1/a^4$ (exactly, given that the particles are non-interacting), and is not the temperature of an interacting hot plasma. Other reasons invalidate this extrapolation, however. Since we took $\Gamma \sim T$, we have $H \sim T$, which means that the mean number of particles per horizon volume is actually of order one. Thus one would already expect at this temperature that quantum effects may become important. Indeed, as we will discuss at greater length in section 8, effects of particle creation in the time-dependent background actually become dominant at this scale, and mean that the purely "classical" model used in the extrapolation breaks down.

Note that the conclusion we drew about bargogenesis in a minimal $SU(5)$, which implicitly supposed that a temperature of order the GUT scale was reached, is not necessarily invalidated by these remarks, although one must of course be careful to check consistency with them. We will discuss, for example, in section 6.2 a model in which the phase of kination starts at a scale well below T_{eq} in Eq. (60) and ends before the nucleosynthesis scale, but in which the reheating temperature after inflation in the early Universe could be near the GUT scale. We will discuss in section 8, on the other hand, a class of models in which the kination phase begins at a temperature of order T_{eq} by construction. In such models indeed these GUT scale baryogenesis scenarios are certainly not viable. On the other hand it may be considered as an interesting features of such models that they never access the GUT scale and therefore never produce any problematic relics associated to the symmetry breaking at that scale.

5.2 A self-tuning scalar component

The second kind of modification of standard FRW cosmology without a scalar field which we consider is one which is directly suggested by the existence of the attractor solutions for the simple exponential potential in which the scalar field simply "locks in" to the same scaling as the other component (radiation or matter), contributing a fraction of the energy density completely specified by the single parameter λ through Eq. (28) above. In particular, if we consider the case that $\lambda > 2$, an attractor exists in both the radiation and matter dominated era so that one expects approximately

$$\begin{aligned} \Omega_\phi(a) &= \frac{4}{\lambda^2} \qquad a < a_{eq} \\ &= \frac{3}{\lambda^2} \qquad a > a_{eq} \end{aligned} \tag{61}$$

The transition between the two solutions around the time of equality ($a = a_{eq}$) occurs very efficiently in a few expansion times (and in a completely detemined way) as seen in Figure 3, which shows the exact evolution found numerically for a range of values λ corresponding to $\Omega_\phi \sim 1$. Following [32] we refer to this kind of model as that of a "self-tuning" scalar field to highlight this simple behaviour of the scalar field in following whichever of the other components (radiation, then matter) dominates.

Note that the potential can always be written, by an appropriate redefinition of the origin of the field, as

$$V(\phi) = M_P^4 e^{-\lambda \frac{\phi}{M_P}} \tag{62}$$

so that there is no new mass scale introduced to cosmology with this scalar potential (λ will be a number of order unity). Further (see discussion in [33]) such potentials arise quite generically (at least in form) in

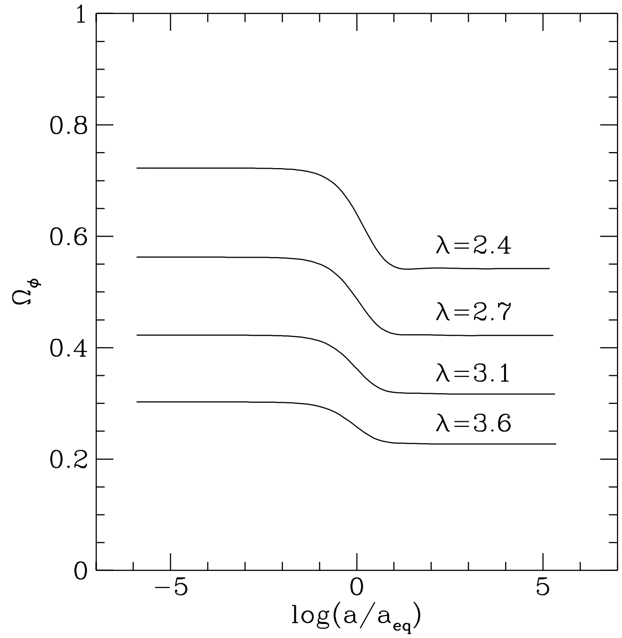

Figure 3: The evolution of the energy density in a scalar field with a simple exponential (exponent λ) as a function of red-shift around the time of equality between matter and radiation.

Particle Physics models. The fact that the cosmology is specified by this single parameter λ and involves no new energy scale is the particularly attractive feature which led us to study it in considerable detail in [32, 33]. Previously the existence of these very particular solutions for an exponential potential had been noted [22, 23], but they had been assumed to be of no particular interest, because of nucleosynthesis constraints which force $4/\lambda^2 \leq \Omega_\phi^N$. This in fact assumes two things, both of which are incorrect: (i) an energy density compatible with nucleosynthesis constrints in a scalar field can have no appreciable effect cosmologically, and (ii) the attractor must be assumed already to be established at nucleosynthesis. The first is not true because the scalar field acts for a long time - right through the radiation and matter dominated era. Its smallness is compensated for by the long time for which it acts. It was in fact the second assumption which originally motivated the investigation - as we will discuss further below, in the context of models with a period of kination prior to nucleosynthesis realized with the exponential potential it is quite natural to expect the attractor solution to be reached well after nucleosynthesis.

The observational interest of this kind of model with a small contribution from a scalar field is primarily in the problem of structure formation: Given that it simply mimics the matter component at late time, homogeneous cosmology (i.e. "classical" cosmological tests) will be unable to distinguish it from ordinary matter. A full discussion of structure formation in this model, in the linear regime, is given in [33]. This involves a simple generalisation of the standard treatment of a CDM cosmology, with the modified homogeneous cosmology as well as the perturbation to the scalar field itself.

The main modifications can be understood as follows: The cosmology resembles alot a mixed dark matter (MDM) cosmology, in which a fraction of the matter density (typically $\Omega_{HDM} \sim 0.2$) is put in a hot neutrino component. This modifies both the background cosmology and the perturbations: it changes the expansion rate as a function of red-shift (and therefore the precise balance between the cold matter and radiation), and also adds the effect of its own perturbations which grow much less efficiently than the CDM. With the "self-tuning" scalar field we have qualitatively the same two effects, although the details are slightly different. While the neutrino component scales like the radiation until considerably after equivalence, changing to matter scaling at a time determined by its mass, the scalar field follows tightly the scaling of the dominant component. This leads to an increase in the expansion rate around decoupling as a function of red-shift relative to either CDM or MDM. This results in a shift to a smaller angular size today of the sound horizon at decoupling, and thus a shift towards larger l of the Doppler peaks in the CMBR. The other effect, relative to CDM, is a slight increase in amplitude of these peaks. In Figure 4 is shown the effect of adding a given fraction of energy in scalar field to a standard CDM model, with a standard flat adiabatic spectrum of initial perturbations.

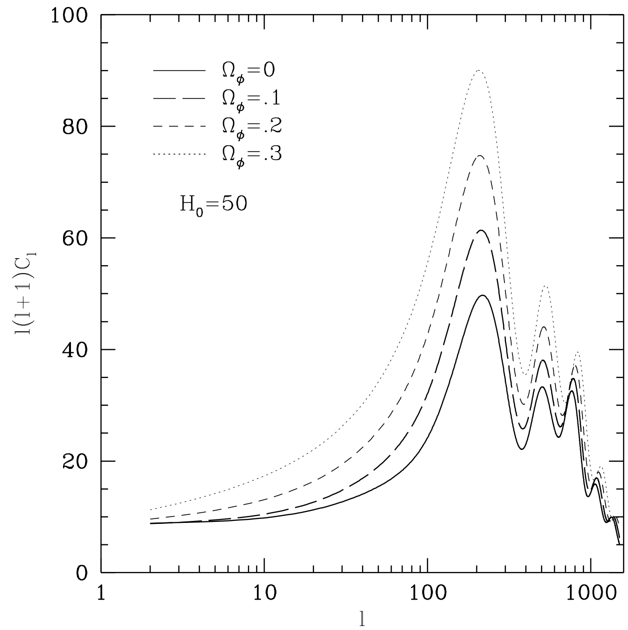

Figure 4: The C_l spectrum of fluctuations in the CMBR when a CDM cosmology is modified by the addition of a "self-tuning" scalar field, for different values of Ω_ϕ (given in the matter era).

As mentioned earlier in section 4.3 the addition of a scalar field component has a direct effect on the growth of perturbations in the CDM, which is described in linear theory by Eq. (47). The perturbations in the scalar field are themselves effectively completely massless[6] and decay from the time when they enter the horizon until today. This is a much stronger effect than in MDM, in which the neutrino perturbations in a given mode only decay for a part of the time they are inside the horizon: the neutrinos, at the later times when they are non-relativistic, have a finite free-streaming scale (which decreases with time) above which clustering can proceed. The qualitative effect of the damping in the scalar perturbations can be understood simply by neglecting them in Eq. (47), and solving for the evolution of the CDM perturbations δ_m in the matter era, with $\Omega_r = 0$ and Ω_m replaced by its attractor value. One finds that there is a growing solution

$$\delta_m \propto \eta^{2+(5/2)[-1+\sqrt{1-(24/25)\Omega_\phi}]} \tag{63}$$

where η is the conformal time. The effect is therefore simply to reduce the exponent of this solution as we increase Ω_{phi}, relative to the usual growth $\propto \eta^2$ in pure CDM. This leads to a suppression in the power today

[6]It is implied by the background solution that the effective mass in the potential is of order the horizon scale at any time. We will discuss this very important constraint on these theories in section 9 below.

at small scales relative to CDM, an effect which has been argued to make such models much more consistent with observations of galaxy and cluster distributions. The very considerable assumptions involved in connecting this result in linear theory to observations at scales where the dynamics of gravitational collapse is highly non-linear certainly should make us very cautious about this conclusion.

Note that of course this cosmology cannot satisfy constraints coming from supernovae, and is no longer considered in itself as a viable variant on CDM. However many quintessence models (see below) incorporate a phase like the "self-tuning" one of these models, keeping its elegant property of independence of initial which making some change to modify its late time behaviour.

5.3 Acceleration at recent epochs - Quintessence

The third model is one which has been much studied in the last few years (under the name "quintessence" [34]) precisely because it provides a possible modification of the standard cosmology which can explain (a posteriori) observations from supernovae. In the late eighties and early nineties models of this kind were proposed by various authors [22, 23, 35, 36] motivated by the "age problem" i.e. the incompatibility between the high measured value of the Hubble constant and the age of astrophysical objects (in particular, globular clusters). In this case (it is perhaps healthy to recall!) the problem was resolved not by the theoretical models but by a reassessment of the empirical constraints. In particular a re-calibration of the distance of globular clusters led to a correction to the estimate of their age, which resolved the incompatibility [37].

The essential element of these models is simply a period, beginning at recent epochs, of domination by a homogeneous mode of a scalar field rolling in its potential. The equation of state when the scalar energy dominates corresponds to an accelerating Universe i.e. $6\xi < 2$ ($w < -1/3$). More specifically, one is of course specifically interested in models with sufficient recent acceleration to explain the Sn observations. For a constant equation of state as we discussed in section 4 this means $\xi \leq 0.2$ ($w < -0.6$).

A completely trivial example which satisfies this criterion is

$$V(\phi) = (0.7)\rho_o \tag{64}$$

where $\rho_o = 3M_P^2 H_o^2$ is the Universe today (assuming flatness). Since the potential is exactly flat we know that, at all times,

$$\frac{1}{2}\dot{\phi}^2 \propto \frac{1}{a^6}. \tag{65}$$

Given that the Universe was not dominated by the kinetic energy of this field at nucleosynthesis, the energy in the field therefore behaves exactly as a cosmological constant with $w = -1$ and $\Omega_\phi^o = 0.7$.

In fact of course this example has a triviality in it which we want to exclude. The only physical context in which the overall zero point of a potential has any significance is through its coupling to gravity, so that what we have here is equivalent to a cosmological constant (along with a field without a potential whose energy relaxes away as $1/a^6$). With a model like this therefore one does nothing more than dress the cosmological constant problem up in another form.

For a quintessence model to have any interest the energy in the scalar field must not be equivalent to a cosmological constant term. To ensure this we assume

$$V(\phi_{min}) + \Lambda = 0 \tag{66}$$

where ϕ_{min} is the minimum of the scalar potential, or, completely equivalently,

$$V(\phi_{min}) = 0 \qquad \Lambda = 0 \tag{67}$$

since we have the freedom to redefine the potential by an overall constant absorbed into the cosmological constant Λ.

Thus in quintessence models *it is supposed that the cosmological constant problem is solved*, so that the energy in the real vacuum is exactly zero. The scalar energy giving rise to the acceleration is associated with the relaxation of the Universe towards this minimum. Note that we do not require the value of ϕ_{min} to be finite[7]. In fact in many models (see section 7 below) one considers potentials which "run away" to their zero minimum as the field runs to infinity. (In particular both the power-law and inverse power-law potentials, whose implementation as quintessence models are discussed below, are of this kind.)

One can of course satisfy the condition Eq. (67) while allowing non-trivial local minima at positive values of the potential. If the dynamics involved in the solution used to produce the acceleration today involves simply the relaxation to such a non-zero minimum, so that the energy is strongly dominated by this potential energy today, one is again essentially in the case like the trivial one above. Another trivial construction can be obtained by satisfying the condition Eq. (67) by multiplying the potential in (64) by a monotonic function going from $+1$ to zero as the field ϕ goes from $-\infty$ to $+\infty$ e.g. multiply the trivial potential in (64) by $(1 - \tanh \phi/M)/2$, where M is some mass scale. If the accelerating solution has the field in the region $\phi << M$, one is once again essentially in the same case[8].

In both these cases the equation of state will be essentially that of a cosmological constant. To exclude cases like this from consideration what we want is that the equation of state be significantly different from this, which is just the same thing as saying that contribution from the kinetic energy of the scalar field be non-negligible as a fraction of the scalar energy, and therefore the total energy.

We can thus define quintessence as *a scalar field rolling today in a potential which vanishes at its minimum, and dominating the Universe at recent epochs with an equation of state corresponding to an accelerating Universe*. What one means by "rolling today" is not just $\dot{\phi} \neq 0$, but requires more precisely that $H_o^2 M_P^2 \sim \dot{\phi}^2$ or

$$\dot{\phi} H_o^{-1} \sim M_P \tag{68}$$

Given that the time scale for the evolution of the field will often naturally be just H_o^{-1} itself (or closely related to it), we will typically have $\phi \sim M_P$. Indeed we have seen in the specific case we have considered of exponential potentials that the scale for the field and its variation over cosmological time was indeed M_P. A counterexample are the cases of fields oscillating about their minima where the time scale for variation is set by the curvature on the potential and not by the expansion rate, but for the simple polynomials we considered this only allows a scaling $1/a^n$ with $n > 3$. For rolling fields one can also write down counterexamples, but they are of the type we have discussed which are effectively indistinguishable from a cosmological constant.

6 Model Building and Fine Tuning in Scalar Field Models

We have already come close to touching on a question of central importance in model-building which is that of fine-tuning. We attempt to discuss it now in a systematic manner.

As we have remarked one can in principle simply design a potential to give the behaviour desired of the scalar field. The question of fine-tuning emerges when one considers exactly how "natural" the proposed potential and dynamics is. In particular it can in principle be divided into two parts:

• **Fine tuning of potential parameters**: Are the parameters (masses, couplings) which are required in the proposed potential tuned from the point of view of Particle Physics?

[7] The condition of Eq. (67) should more accurately be written as $Inf[V(\phi)] = 0$.
[8] From the flatness/slow-roll condition this requires $M \geq M_P$.

• **Fine tuning of initial conditions**: For the evolution envisaged in the given potential are very special initial conditions required in the early Universe to give the scalar field energy the desired cosmological history? If so, are the required initial conditions natural?

In the context of model building for quintessence, as we will discuss at some length below, fine-tuning problems are usually encountered in the first form. In this context they are referred to as the "coincidence problem". The "coincidence" is that the scales in the potential have to be such that the scalar energy comes to dominate today, and not at some earlier time. In this context the fact we have just considered, that in most models $\phi \sim M_P$, is of importance. In Particle Physics this has the very specific meaning that we are treating condensates of order the Planck energy, and thus inevitably dealing with the (empirically unknown and probably untestable!) physics at that scale, which would seem to make it a hopelessly speculative endeavour to constrain the potential. A more optimistic view is that the cosmological effect of such fields - in particular as envisaged in the models we have been discussing - would give, if observed, a small "window" on the physics at these fundamental scales. The "coincidence" problem in the context of these theories concerns how we get small mass scale (or scales) which are simultaneously required by these models out of this physics.

To address the second question for any model one must of course be clear about what one means by the word "natural" in this context. Before going further it is a good idea that we clarify this. The answer one gives to this question depends on what framework one works in to describe the beginning of the FRW phase of the Big Bang model. For the moment we will consider a criterion of naturalness, based on the idea that whatever happens we might expect a rough equipartition between the energy in the scalar field and the radiation at the beginning of the classical FRW phase i.e.

$$\frac{1}{2}\dot{\phi}^2 \sim V(\phi) \sim T_{rh}^4 \tag{69}$$

where T_{rh} is the "reheat" temperature[9]. In the specific context of a standard inflationary model (driven by another scalar field, the inflaton) this will be satisfied, assuming that the potential energy $V(\phi)$ is comparable to that in the inflaton at the beginning of inflation, and that the potential itself is not inflationary at this time, so that it can roll during the reheating phase. Our discussion below of the explicit construction of kination models will lead us to discuss an alternative model of matching from inflation to the radiation dominated FRW phase in which the very different initial conditions required for this case become "natural".

Despite first appearance the two fine-tuning problems, of potential parameters and initial conditions, are in fact closely related, and can often be recast from one form into the other, as we now illustrate with various examples.

6.1 Quintessence

Consider first the case of quintessence. Being essentially just inflation, only at much lower energy scales, a first guess might be to try to realize it a simple generic quadratic potential as in chaotic inflation scenarios. We take $V(\phi) = m^2\phi^2$, and applying the "slow-roll" criterion for inflation, which we have discussed in terms of the scaling solutions in exponential potentials above, we need, $M_P V'/V < 1$ i.e. $\phi > 2M_P$. Thus we require the field to lie today anywhere farther up the potential than $\phi \sim 2M_P$. Further we need the energy in the corresponding part of the potential to correspond to the energy density of the Universe today ($\rho_o \sim H_o^2 M_P^2$). If we suppose the field to lie at $\phi = xM_P$ today ($x > 2$), the mass must be $m^2 \sim H_o^2/x^2$, or at most $m \sim 10^{-33}$ eV! Further the initial conditions must be such as to leave the field at this precise part

[9] We do not include here, for clarity, the factor g_* which one would naturally put in front of T_{rh}^4.

of the potential today. It is not difficult to see this implies that the field must essentially lie at this point in the potential from early times. The reason is that at larger values the potential is even flatter, so that any reasonable amount of kinetic energy damps away rapidly (with $1/a^6$). The field then simply sits where it is until the epoch at which it begins to dominate. If it lies further down the potential initially it will roll down and eventually oscillate today in a matter type mode. Only some very special initial conditions making it roll back up the potential could possibly give the desired result.

We can do better by a more astute choice of potential. Consider again the simple exponential we have discussed. For $\lambda^2 < 1.2$ we have attractor solutions giving rise to acceleration and equation of state $w < -0.6$. What is more we have noted that the potential can always be written in the form of Eq. (62) so that there is no fine-tuning in the potential. To make use of this potential as quintessence, we have to impose however very special initial conditions: the problem is that the attractor is an attractor to the pure scalar field cosmology in which $\rho_n \to 0$. We must have initial conditions which are precisely those which mean we are just entering the attractor today so that $\rho_\phi \approx \rho_n$. Like in the previous case what this means in fact is that the field must lie initially at $\phi \approx \phi_o$, with

$$M_P^4 e^{-\lambda \frac{\phi_o}{M_P}} \approx H_o^2 M_P^2 \tag{70}$$

The attractor behaviour means that if we have any initial conditions resembling those in Eq. (69) we will reach the attractor rapidly (in a few expansion times). If the field is initially further down the potential, it will, once it has got rid of any excess of initial kinetic energy (by $1/a^6$ scaling) essentially sit at a fixed value of the field: The fact that scalar field is dominated means that it is much more damped than in its own inflationary solution. It therefore "sits and waits" for the dominant component to catch up with it and then enters the attractor[10]. To enter the attractor today we have to put the field in the early Universe at the point at which the initial energy density is very close to that today. In this model we have therefore no tuning of our potential parameters, but fine-tuning of our initial conditions. How bad is this fine-tuning? In terms of the field one requires $\phi_o \sim 300 M_P$ which might not look so bad. Physically however the tuning is just the same as in the case of the simple quadratic potential: the energy density in the quintessence field at reheating must essentially be that today.

Making further use of our previous discussion of exponentials potentials it is easy to see how we can modify this potential to avoid this tuning of the initial energy density. Consider instead the potential

$$\begin{aligned} V(\phi) &= M^4 e^{-\lambda_1 \phi/M_P} \qquad \phi < 0 \\ &= M^4\, e^{-\lambda_2 \phi/M_P} \qquad \phi > 0 \end{aligned} \tag{71}$$

where $M^4 \sim \rho_o$, and λ_1 chosen sufficiently large that the attractor solution given by equation (28) exists in the radiation dominated era and satisfies the nucleosynthesis bound ($\lambda_1 > 5 - 10$), and $\lambda_2 < 1.1$ to give the quintessence solution around the present era (since $\xi = \lambda^2 < 1.2$ i.e. $w < -0.6$). Given the attractor behaviour of the potential at larger values of the field we can now set the initial conditions in a very wide range, including certainly the "natural choice" Eq. (69).

The price of resolving the problem for the initial conditions is however clearly paid in the potential, where the scale $M \sim (\rho_o)^{1/4}$ now appears explicitly. Even if we make the potential look less constructed with a smooth function λ interpolating between the values λ_1 and λ_2, *we must inevitably introduce a characteristic scale in the potential at which λ decreases to values giving inflationary solutions, and this scale must simply be ρ_o*. This is the *coincidence problem.*

This simple example actually captures an essential point in the discussion of the fine-tuning of initial conditions in the literature on quintessence, and which we will discuss further below in section 7. There

[10] A more detailed discussion of this dynamics is given in section 7 below.

is, to our knowledge, no way around the problem of fine-tuning in these models[11] ; at best the tuning problem can be moved from one form to another, or made less apparently problematic for Particle Physics in certain potentials. Consider in particular the inverse power-law potential mentioned in section 3 for its capacity to give scaling solutions in the regime $\rho_n >> \rho_\phi$. These same solutions have an attractor behaviour which, as will be discussed further in section 7 can lead to an insensitivity to initial conditions similar to that obtained with the exponential potential. To have quintessence one requires that the scalar field be inflationary at the time we exit from this phase i.e. when $\rho_\phi \sim \rho_o$. Writing the potential in the form

$$V(\phi) = M^4 \left(\frac{M}{\phi}\right)^m \tag{72}$$

where M is some mass scale to be determined, the condition is that the field lie today at $\phi_o = x_o m M_P$ with $x_o > 1$, and thus that

$$M \approx \left(\frac{\rho_o}{M_P^4}\right)^{\frac{1}{4+m}} (x_o m)^{\frac{m}{4+m}} M_P \tag{73}$$

Given the dependence in the exponent, for modest values of m one can increase the scale M in the potential by orders of magnitude from $M \sim \rho_o^{1/4}$ (as in the exponential potential). Therefore, relative to the case of the exponential potential, the hierarchy between the scales M which appears in the potential and the Planck scale can be enormously reduced. The "price" is in the addition of the extra parameter m, with the associated rather unusual form of the potential, in particular for larger m [12]. Note that the fact that $M >> \rho_o^{1/4}$ does not mean that the coincidence problem is (or is not) resolved: M is not the scale where the potential becomes inflationary, rather this scale is still at $V(\phi) \sim \rho_o^{1/4}$, at least if $x_o \sim 1$. It has been suggested that the existence of the scaling solutions in this potential may help to relieve the coincidence problem, which we cannot discount as we could in principle have $x_o >> 1$. We explain in section 7 why this is not the case, and argue that the coincidence problem is essentially equally present in this potential.

This example illustrates well the point that what form we take the fine-tuning problem to be in - in the potential or in the initial conditions - is crucial when it comes to understanding its possible resolution. If we have the fine-tuning in the potential only it is solely a problem for Particle Physics to resolve, by coming up with a good explanation of the origin of the required small parameters. In this context the fact that one can move very considerably the explicit scale M which controls the coincidence may be of fundamental importance. For the inverse power law we can have, for $m = 4$, the scale $M \sim$ TeV, a scale naturally associated with supersymmetry breaking in many particle physics models incorporating this symmetry [13]. This kind of model potential has for this reason provoked considerable interest [38]. It is often understood to be a particularly well motivated potential because of the existence of its scaling, or "tracking" solutions. We will explain below that this kind of behaviour is not an essential element of avoiding fine-tuning of initial conditions, and from this point of view this quintessence model is no better (or worse) than a very wide class of such models.

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[11] We mean here of course models of the type we are treating. Some of the many attempts to resolve the fine-tuning problem will be discussed briefly below in section 10.

[12] In fact m cannot be too large (> 6) for another reason which we will discuss below in section 7: In the scaling solution the equation of state is far from the accelerating one of quintessence, too far in fact to give a sufficiently accelerating equation of state even by the time when $\Omega_\phi \sim 0.8$.

[13] For $m = 4$, however, the scaling solution is never attained from the "natural" initial conditions of Eq. (69), so that the quintessence equation of state is more sensitive to initial conditions. See discussion in section 7.

6.2 Kination

Let us turn now to kination models, in which a kinetic energy dominated mode dominates prior to nucleosynthesis. In particular we are interested in the case discussed in which this phase includes the electroweak phase at temperature T_{ew} and lasts until shortly before nucleosynthesis i.e. the temperature T_e (at which the scalar and radiation densities are equal) is not far above its allowed lower value of $\sim 5MeV$.

A simple way of realizing this scenario is clearly again with a steep exponential, with $\lambda \geq \sqrt{6}$. We saw that, provided the scalar field is dominant, the energy density in this case has an attractor scaling as $1/a^6$. This however is not enough to ensure that our initial conditions are not fine-tuned, because this scalar dominated attractor is not the global attractor, which is instead the "self-tuning" one with $\Omega_\phi = 4/\lambda^2$ (in the radiation dominated phase). Thus if we start at $T_{rh} >> T_{ew}$ with the "natural" initial conditions of Eq. (69), it is this solution we will be drawn to rapidly and not the scalar dominated one. We thus require initial conditions in which the scalar field energy dominates over the radiation in the approximate ratio

$$\frac{\rho_\phi(T_{rh})}{\rho_{rad}(T_{rh})} \sim \left(\frac{T_{rh}}{T_e}\right)^2. \tag{74}$$

This is approximate not just because it neglects possible decouplings of species in the range of temperatures from T_{rh} to T_e, but also because it assumes that we have perfect $1/a^6$ scaling for the scalar field. In practice there will be a short transcient of a few expansion times (depending on the precise initial distribution of the scalar energy between kinetic energy and potential energy) during which the scaling will be slower.

We have here a fine-tuning of initial conditions which is exactly the opposite of that we encountered in the case of quintessence with the simple exponential potential. Just as in that case, the fine-tuning can be moved here from the initial conditions into the potential. What we require now is a potential like that in Eq. (71) but now with a transition around the scale M (to be determined) from $\lambda_1 < 2$ to $\lambda_2 > \sqrt{6}$. If we suppose that we start after standard inflation with an initial condition like that in Eq. (69), the field will lie in the first part of the potential and be drawn towards the scalar field dominated attractor with $\rho_\phi \propto a^{-\lambda_1^2}$. When the field arrives at $\phi = o$, where the energy density is M^4, we will have

$$\frac{\rho_\phi}{\rho_{rad}} \approx \frac{M^4}{T_o^4} \approx \left(\frac{T_{rh}}{T_o}\right)^{4-\lambda_1^2} \qquad \text{or} \qquad T_o \approx \left(\frac{M}{T_{rh}}\right)^{\frac{4}{\lambda_1^2}} T_{rh} \tag{75}$$

where T_o is the temperature of the radiation at this time. The scalar energy then rapidly rolls into the $1/a^6$ in the steeper potential, until it catches up with the radiation at temperature T_e, in terms of which T_o is given as

$$T_o \approx \left(\frac{T_e}{T_{rh}}\right)^{\frac{2}{6-\lambda_1^2}} T_{rh}. \tag{76}$$

If we require T_e to be near its lower limit ($\sim$ 5MeV), and $T_o > T_{ew}$ (so that kination begins before the electroweak scale), this places an upper bound on the value of λ_1^2 given T_{rh}. Taking $T_{rh} \sim 10^{15}$GeV, we can satisfy the bound for $\lambda_1^2 = 3$ (i.e. for a matter type scaling in the steeper part of the potential). We can then determine the corresponding value of the mass scale M as

$$M \approx \left(\frac{T_e}{T_o}\right)^{\frac{1}{2}} T_o \approx \left(\frac{T_e}{T_{rh}}\right)^{\frac{\lambda_1^2}{2(6-\lambda_1^2)}} T_{rh}. \tag{77}$$

which for $T_{rh} \sim 10^{15}$GeV and $\lambda_1^2 = 3$ corresponds to $M \sim 10^6$GeV, increasing to larger scales as λ_1 decreases more towards inflationary values. In principle, since $M \approx \sqrt{T_e T_o}$, the smallest value of M required to get $T > T_{ew}$(i.e. $T >$ 100GeV) is $M \sim$ 30TeV, independently of the reheat temperature.

Like in the case of quintessence therefore we can remove the fine-tuning of the initial conditions required in the simple exponential but at the price of introducing a new mass scale in the potential, characterising where it changes shape (going from flatter to a steeper form). In this case this scale can essentially lie anywhere a little above the TeV scale, rather than the tight requirement $M \approx 10^{-3}$ eV in the case of quintessence with the same exponential form. Even if the constraint is much weaker one can say analagously that there is a "coincidence" problem in the model. The temperature T_e (of the transition to radiation domination) is given by

$$T_e \approx \left(\frac{M}{T_{rh}}\right)^{\frac{2(6-\lambda_1^2)}{(8-\lambda_1^2)}} T_{rh}. \tag{78}$$

By construction this temperature will always lie below the reheat temperature after standard inflation (the exponent in Eq. (78) is greater than unity for $\lambda_1 < 2$, and we must have $M < T_{rh}$). For the scenario we have discussed (of kination at the electroweak scale) the coincidence is that this temperature T_e happens to be close to the nucleosynthesis scale.

Note that we took only $\lambda_2 > \sqrt{6}$, while a priori one might expect that it is necessary to take $\lambda_2 > 8$ to satisfy the nucleosynthesis constraint in the "self-tuning" attractor in this part of the potential. We will see in the next section why the weaker constraint is in fact sufficient in this case.

An alternative way in which we can realize kination is with the oscillatory mode of a polynomial potential ϕ^n, which as described in section 3 leads to an energy density scaling approximately as $a^{-\frac{6n}{n+2}}$, which for $n > 4$ scales faster than radiation. To be specific let us take the potential in Eq. (35), with M chosen so that $\lambda_n \sim 1$ for some $n >> 4$, and all other λ sufficiently small that they can be neglected. Such a simple power law potential is inflationary for $\phi \geq nM_P$ and oscillates once it reaches smaller field values. One can thus in this case also implement a scenario like the preceeding one, as the scalar field can first inflate for a period, "cooling" the radiation and then enter the oscillating mode. This however involves a greater dependence on initial conditions that in the previous case. There the temperature T_e was well defined given the potential and T_{rh}, while here we will need to choose delicately the initial condition at the end of inflation. Suppose the field lies at $\phi = xnM_P$ after inflation when the temperature is T_{rh}. For the natural initial conditions as in Eq. (69) we have the relation

$$M \approx \left(\frac{xnT_{rh}}{M_P}\right)^{\frac{n}{n-4}} M_P \tag{79}$$

Then once x is chosen it will determine both T_e and T_o. For $x << 1$ the oscillating mode begins at once, and $T_{rh} \sim T_e \sim T_o$, while for $x >> 1$ there will be a long period of inflation driving T_e arbitrarily small. Only for a very specific choice of initial condition will we get the required behaviour. Thus here we have a less tuned potential (arguably) but more fine-tuning of initial conditions (albeit in a range where they are "natural" in the sense of Eq. (69). In fact in the example of the steepening exponential potential we managed to effectively move the tuning here of the initial condition parameter x to that of the potential parameter λ_1. Another problem with such oscillatory models compared to damped roll models is that one must watch out for resonance effects which can make an oscillating condensate unstable to decay into its own excitations (see [26] for a discussion of the constraints imposed on the couplings by this consideration).

These models producing kination from "natural" initial conditions after ordinary inflation have involved putting an additional phase which drives the scalar field dominant between the end of ordinary inflation (around T_{rh}) and the onset of kination (at T_o). We will discuss in section 8 below an alternative scenario for the transition from inflation to radiation in which the existence of a phase of kination is an intrinsic part of the mechanism.

6.3 Self-tuning models

The second model above - of the "self-tuning" scalar field - is, as we discussed, particularly motivated by the precise feature that it does not have problems with tuning. The potential is the simple potential involving no scale other than M_P, and couplings of order unity. And the solution used is an attractor solution for the set of coupled scalar field-matter/radiation equations, thus ensuring independence of initial conditions.

There is one caveat. In studying these models we are interested in the effects on structure formation, which only depends (up to initial conditions on the fluctuations) on the fact that the attractor is supposed to apply from a time when the Universe has cooled to at least a little above the eV scale. The study in [33] found that the effect of this component was similar to that of the hot component in standard MDM cosmologies with an fractional density in the matter era $\Omega_\phi \sim 0.1$ ($\lambda \sim 5$). This is however more than allowed by recent tighter nucleosynthesis bounds which, as discussed in section 4, give $\Omega_\phi \leq 0.045$ ($\lambda > 10$). Thus, if one accepts these tighter constraints, one must assume for consistency that the attractor is entered *after* nucleosynthesis, and sometime before the era of structure formation. While this changes nothing in the predictions of these models, it undoes a significant part of what makes them a case apart in cosmological models with scalar fields.

Without changing the potential the only alternative is apparently to take the (completely "unnatural") initial condition that makes the initial potential energy in the scalar field somewhere below the nucleosynthesis scale. Alternatively one can "mend" this fine-tuning problem by modifying the potential to be slightly steeper (large λ) at smaller field values, the price being again the introduction of a characteristic mass scale in the potential somewhere in the range of MeV-eV.

An interesting alternative is given if one considers precisely the kind of kination model we have discussed. When we say that the simple exponential, with $\lambda > 2$, will enter the attractor before nucleosynthesis, this is predicated on the assumption that the initial conditions are not so far from this attractor (i.e. that the transcient behaviour is short), as will be the case for example if they are the "natural" ones of Eq. (69). If, however, the kinetic energy dominates completely over the potential energy at some time before nucleosynthesis, this will not be the case.

The reason is the following. For the simple exponential the attractors in the case of scalar field dominance have $\xi = 1 - \lambda^2/6$, for $\lambda < \sqrt{6}$, while for $\lambda > \sqrt{6}$ the kinetic energy becomes arbitrarily dominant as the field evolves, with $\xi \to 0$. Why this happens can be understood easily. If the kinetic energy dominates the total energy at some time t_o we have $\frac{1}{2}\dot{\phi}^2 \propto 1/a^6$, and $a \propto t^{\frac{1}{3}}$. Therefore one can integrate to find

$$\phi(t) = \phi_o + \dot{\phi}_o t_o \ln(t/t_o). \tag{80}$$

We also have that $H_o^2 \approx \dot{\phi}_o^2/6M_P^2$, and $H_o t_o = 1/3$, so that

$$\phi(t) = \phi_o + \sqrt{\frac{2}{3}} M_P \ln(t/t_o). \tag{81}$$

Now the ratio of potential energy to the kinetic energy ($= \frac{1-\xi}{\xi}$) is

$$\frac{M_P^4 e^{-\lambda(\phi_o + \sqrt{\frac{2}{3}} M_P \ln(t/t_o))/M_P}}{\frac{1}{2}\dot{\phi}_o^2 (t_o/t)^2} = \frac{V(\phi_o)}{\frac{1}{2}\dot{\phi}_o^2} \left(\frac{a}{a_o}\right)^{6(1-\frac{\lambda}{\sqrt{6}})} \tag{82}$$

Therefore if $\lambda > \sqrt{6}$ this quantity (which is just $1 - \xi$ for $\xi \approx 1$) is driven small extremely rapidly by the roll down the potential. For example with $\lambda = 5$ and a period of kination lasting over a red-shift range of 10^r, the ratio of the energies at the end of this time ia about 10^{-6r}.

To reach the attractor with the radiation or matter however this same parameter must grow to be of order one (0.5 in radiation domination, 1 in matter domination). The best the potential energy can do is stay fixed (the field cannot roll up the hill!) while the kinetic energy can decrease, but at most as fast as $1/a^6$. Thus the total energy in the scalar field must continue to decrease for a long time until its kinetic energy "catches up" with the potential energy. Only then can the scalar energy begin to grow back, scaling slower than the dominant component, to eventually join the late-time attractor. Thus with a steep exponential ($\lambda > \sqrt{6}$) a domination at early times by its energy over the radiation will always lead to a (potentially very long) transcient period of "undershoot", in which the scalar energy is completely sub-dominant with respect to the dominant one, which asymptotically it will approach in the attractor solution with $\Omega_\phi = n/\lambda^2$. In Figure 5 the results of a numerical integration of the equation of motions for such case, with $\lambda = 4$, and the scalar field initially dominant, illustrate the behaviour we have described.

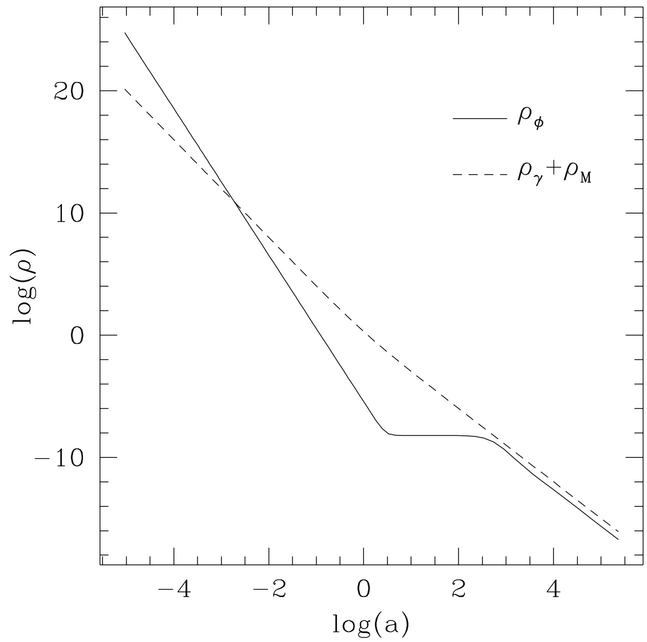

Figure 5: The evolution of the energy density in a scalar field (solid line) and radiation-matter fluid (dashed line) for a simple 'steep' exponential potential ($\lambda = 4$) as a function of red-shift, when the scalar field starts as the dominant energy component. $a = 1$ is the time of matter-radiation equality, explaining the change in slope in the dashed line.

We will return to give a more qualitative analysis of how the entry time to the attractor depends on the early Universe history in the context of the specific alternative model for the transition to radiation we consider in which the phase of kination is a central feature. Before doing so we return to quintessence models, to show how the same kind of steep potentials required for kination are very naturally required also in this context.

7 Quintessence models without fine-tuning of initial conditions

In this section we consider model building in quintessence in further detail. After a discussion of various potentials which have been proposed in the literature, we give simple criteria for the construction of a quintessence potential which does not require fine-tuning of initial conditions i.e. which will give a consistent early Universe cosmology and quintessence at the current epoch from a large basin of "natural" initial conditions. We note that the attractor behaviour associated with sufficiently steep exponentials, or the "tracking" solutions which can be obtained in steeper inverse power-law potentials, are a very particular way of satisfying these criteria, by giving a much stronger property of complete *independence* of initial conditions. We emphasise that in a much wider class of potentials (given by our criteria) quintessence today results starting from natural initial conditions. The reason is simply that there is a much more general attractor dynamics in play associated with the required late-time inflationary behaviour of the potential, which guarantees that quintessence behaviour is obtained irrespective of whether there are exact or approximate attractors for the dynamics in the early Universe.

It is perhaps worth emphasizing again that while we can find many potentials satisfying the criteria we give, without fine-tuning of initial conditions, this is not *necessarily* the correct way to find the physical resolution of fine-tuning problems in these models. As we have discussed one can easily find potentials without any fine-tuned parameters in the potential, but which require apparently extremely unnatural fine-tuned initial conditions to give quintessence. Putting the problem in the potential makes it one for particle physics alone to resolve, while putting it in the initial conditions makes it primarily a cosmological one. Generally one simply assumes (implicitly) that any reasonable cosmology for the primordial Universe will give initial conditions on the quintessence field like those of Eq. (69). A counterexaple is given by the alternative model of exit from inflation we consider below. And certainly other frameworks for understanding how we get into the FRW phase may give different notions of naturalness. We note that even in the context of standard inflationary models the equipartition assumption of Eq. (69) is far from obvious: after all the quintessence field is a field extremely weakly coupled to the ordinary degrees of freedom, so why should it have a comparable energy density?

With this cautionary note we now continue to discuss the criteria for the construction of simple quintessence potentials which should be free from fine-tuning of initial conditions (over a wide basin including, the "natural" ones of Eq. (69). To arrive at a formulation of these criteria let us consider a collection[14] of potentials for quintessence which have been proposed in the literature:

$$\Lambda^4 \exp[-\lambda(\phi)\phi] \qquad \text{Wetterich} \quad [22] \tag{83}$$

$$\frac{\Lambda^{4+\alpha}}{\phi^\alpha} \qquad \text{Ratra - Peebles} \quad [23] \tag{84}$$

$$\Lambda^4[1+\cos(\phi/f)] \qquad \text{Frieman et al} \quad [36] \tag{85}$$

$$\Lambda^4[\exp(-\alpha\phi)+\exp(-\beta\phi)] \qquad \text{Copeland et al} \quad [39] \tag{86}$$

$$\Lambda^4[(\phi-B)^\alpha + A]\exp(-\lambda\phi) \qquad \text{Albrecht - Skordis} \quad [40] \tag{87}$$

$$\frac{\Lambda^{4+\alpha}}{\phi^\alpha}\exp(\frac{K}{2}\phi^2) \qquad \text{Brax - Martin} \quad [41] \tag{88}$$

$$\Lambda^4[\cosh(\lambda\phi)-1]^p \qquad \text{Sahni - Wang} \quad [42] \tag{89}$$

$$\Lambda^4[1+A\sin(\nu\phi)]\exp(-\lambda\phi)) \qquad \text{Dodelson et al} \quad [43] \tag{90}$$

where the field ϕ is taken to be normalised in units of M_P.

In the first one (83) the exponent is taken to be a function of ϕ decreasing slowly as ϕ increases, in a smoothened version of the two exponent model described in section 6.1. Until nucleosynthesis scales at least $V(\phi) \sim (MeV)^4$ the exponent is larger than that required at that time (i.e. $\lambda > (5-10)$, depending on the exact nucleosynthesis constraint adopted), increasing to reach inflationary values at $V(\phi) \sim \rho_o$, (and certainly at values of the potentials sensibly above this, as the inflationary solution is an attractor which will rapidly lead to scalar field domination). The potential in (86) is a simple variant of this, being essentially just the two exponent model we have discussed, with α large are β small, in exactly the way which makes them satisfy the two conditions (with here $\Lambda^4 \sim \rho_o$).

[14]I am grateful to J.P.Uzan for giving me this particular selection.

It is evident that these potentials show independence of initial conditions, over an extremely wide range around the "natural" initial conditions of Eq. (69), simply because the early time behaviour establishes a single trajectory for the system when it enters its late time phase. Several other of these potentials can be understood in exactly the same way. In (87) the exponent λ is again chosen to give the early time "self-tuning" attractor consistent with nucleosynthesis; the extra polynomial structure in front of the exponential produces a new (non-zero) local minimum at a finite field value. When the field runs towards this region, it finds itself in a flatter region with inflationary solutions around $\phi \sim M_P$. (This is evident because around the finite minimum there is a power law expansion). The potential parameters are chosen so that the potential at this point has height $V(\phi) \sim \rho_o$, and the early time attractor has determined the dynamics in this region also. The potential (89) is identical in how it works - at large values of the field, where the field lies initially, we have approximately simple exponentials with exponent $p\lambda$ chosen large enough to give the "self-tuning" attractor; as one goes towards the minimum there is again an inflationary behaviour, in which the evolution of the system is well determined because of the early time attractor. Thus these potentials are essentially power-law potentials which give inflation matched onto steep exponentials which avoid inflationary solutions at large values of the field, and even better make the evolution uniquely determined.

In summary the essential element of these models is the following: The potential in each case has an asymptotic inflationary attractor, which will be reached asymptotically for almost any initial conditions[15]. What we need for quintessence is that we are just entering this attractor now i.e. that we take a very particular sub-class of transcients to this solution. One way of doing this is by making the initial conditions (on $\dot{\phi}$ and ρ_n) exactly determined when we reach this part of the potential. The steep exponentials at larger values of the potential do this as they "deliver" the field and total energy density to the inflationary region with determined values, independently of their conditions at the real initial time. In all of these potentials, however, one manifestly still has the "coincidence problem", that the energy density of the potential at the beginning of the inflationary region is $\sim \rho_o$.

The inverse power-law potential (84) has been much discussed in the literature on quintessence, in particular following the discussion of it in [44]. One reason is that we have discussed in section 6.1: the mass scale M which appears explicitly in the potential may be a quite reasonable scale from the point of view of particle physics, where indeed these kinds of potentials can appear in certain contexts[38]. The other reason for interest in them is that, as discussed at length in [44], they also show, in certain cases, a great insensitivity to initial conditions, somewhat like that in the exponential models. Further, it has been argued [44] that they may help to relieve the coincidence problem.

As was discussed briefly in section 3 this potential possesses in the radiation or matter dominated era an approximate attractor ("tracker" solution following the nomenclature of [44]) for the system when $\rho_\phi << \rho_n$. It is not evident given the preceeding discussion of exponential potentials why this kind of solution shows an evolution independent of initial conditions. For we have seen there that this property came from the fact that the field arrives in the flat part of the potential today with determinate initial conditions. Here the "tracker" corresponds, as we have described it, to an evolution in which ξ is fixed i.e. the kinetic energy is fixed as a function of the potential energy, but no relation to the value of ρ_n has been given. Indeed, we have noted that these solutions are transcients to the late-time inflationary solution - how could the scaling behaviour of this transcient tell us when the transcient ends? The answer as we will explain now is that it does not. What is important is that the evolution of ρ_n is also regulated in these

[15] The potential (87) has the displeasing property that the minimum one is rolling towards is not the global one: it means that the model effectively has a small cosmological constant term, and correspondingly a parameter range where it is trivially close to a cosmological constant (when the field gets stuck at the local finite height minimum). Future measurements showing w arbitrarily close to -1 could not rule out such a model.

solutions, leading to the fact that the "tracker" also ends at a determinate time, when $\rho_\phi \sim \rho_o$, with a completely determined quintessence solution emerging as it does so. Our discussion of the evolution in this potential however leads us to explain this potential is just an example of a much more generic 'steep-flat' form required to give a late-time entry to a quintessence solution for any "natural" initial conditions. Therefore, while the tracking properties of the simple inverse power law potential are notable, they are not necessary for the avoidance of fine-tuning problems.

For what concerns the coincidence problem, at first sight the "tracker" solutions in this kind of potential do suggest a possible amelioration, in that they give a phase in which the scalar energy can mimic that of the dominant (radiation or matter) component. The scalar field has a dynamics in which at early times it becomes extremely subdominant, but then is pulled into the "tracker", slowly gaining on the dominant component until it comes to dominate (inevitably) itself. One might think then that the energy scale at the scalar field becomes dominant ($\sim \rho_o$) need not necessarily be so strictly connect to the scale at which it becomes inflationary. This is not the case: the "tracker" behaviour is in fact associated with the slightly steeper part of the potential; if one enters these solutions one necessarily exits to scalar field dominance before the potential becomes inflationary. In fact this causes a problem for the use of these solutions at all: they have only a limited time to roll into the flatter part of the potential and can give at very most (for $\Omega_\phi = 0.8$ today) an acceleration $w \approx -0.6$, which is apparently at the limits of compatibility with supernovae observations combined with constraints from CMBR data [7].

Let us first consider the dependence of the late time dynamics on initial conditions, and see why the inverse power law shows in certain cases effective independence of initial conditions. As we discussed in section 3 the behaviour of the energy in a scalar field potential can be understood by comparing it with the simple exponential i.e. by considering the value of $\lambda_{eff} = |M_P V'/V|$. In the inverse power law in Eq. (72) we have $\lambda_{eff} = mM_P/\phi$, so that the field is steep/fast-roll for $\phi << mM_P$, and then inflationary for $\phi > mM_P$. With "natural" initial conditions the field starts (given the normalisation) very far up the potential, where λ_{eff} is extremely large. The field sees an extremely steep potential and rolls away in a kinetic energy dominated mode with $\rho_\phi \approx \frac{1}{2}\dot{\phi}^2 \propto 1/a^6$. Consider the evolution from some time t_o at which the field lies at ϕ_o, with derivative $\dot{\phi}_o$ ($\frac{1}{2}\dot{\phi}_o^2 >> V(\phi_o)$). Given that t_o is necessarily sometime in the matter or radiation dominated epoch, with $a \propto t^{2/3}$ or $a \propto t^{1/2}$ respectively, $\frac{1}{2}\dot{\phi}^2 \propto 1/a^6$ can be integrated to give

$$\phi(t) = \phi_o + 2\dot{\phi}_o t_o [1 - (\frac{t_o}{t})^{\frac{1}{2}}] \qquad \text{radiation dominated} \tag{91}$$

$$= \phi_o + \dot{\phi}_o t_o [1 - \frac{t_o}{t}] \qquad \text{matter dominated} \tag{92}$$

These solutions describe the fact that the roll of the field is extremely damped, compared to the logarithmic solutions we found (Eq. (80)) when the scalar field itself dominates. Indeed the field always has most of its displacement in an expansion time, and asymptotically approaches the value $\dot{\phi}_o t_o$ (or $2\dot{\phi}_o t_o$). To see the corresponding characteristic change in the potential energy during this roll we write the Taylor expansion

$$V(\phi_o + \dot{\phi}_o t_o) = V(\phi_o)\left[1 + \frac{\dot{\phi}_o t_o}{M_P}\frac{M_P V'(\phi_o)}{V(\phi_o)} + \frac{1}{2}(\frac{\dot{\phi}_o t_o}{M_P})^2 \frac{M_P^2 V''(\phi_o)}{V(\phi_o)} + ...\right] \tag{93}$$

Given that the kinetic in the scalar field is a sub-dominant component in the total energy we have $\dot{\phi}_o t_o / M_P << H_o t_o \sim 1$. Once one reaches the flatter part of the potential the dimensionless gradient terms become small, so that when ϕ_o is in this region the field "gets stuck", with its energy changing negligibly. (In the very steep part of the potential the gradient terms are so large that the potential energy decreases faster than kinetic energy, giving precisely the kinetic energy dominated phase we have supposed we are in). The total scalar energy density will continue to evolve as $1/a^6$ until the potential energy has had time to "catch up" with the decreasing kinetic energy i.e. until red-shift a_1 such that

$$\frac{1}{2}\dot{\phi}_o^2\left(\frac{a_o}{a_1}\right)^6 \approx V(\phi_o) \tag{94}$$

where a_o is the redshift at time t_o. At this time what happens depends on the gradient of the field at this point: the field remains "stuck" as implied by Eq. (93) as long as it is true that $\frac{1}{2}\dot{\phi}^2 \propto 1/a^6$, which holds as as long as the gradient term V' is negligible in the equation of motion i.e. given that now $\dot{\phi} \approx \sqrt{V}$, and ($\dot{\phi} \propto 1/a^3$) the condition that we continue in this overdamped regime is

$$\frac{V'}{3H\dot{\phi}} \approx \frac{M_P V'/V}{3\sqrt{\rho_n/V}}\left(\frac{a_1}{a}\right)^3 << 1. \tag{95}$$

Since the numerator by assumption is not too large, and ρ_n dominates, this remains valid until the term which is changing ($\propto \rho_n/a^3$) decreases sufficiently that the condition breaks down. Until this time the energy density in the scalar field is still effectively constant (with $\rho_\phi \approx V(\phi_o)$. Subsequently the field can begin to roll again as it sees its gradient, and the kinetic energy increases again. It cannot become too dominant however because this brings us again into the overdamped regime. There is thus a kind of self-regulating mechanism in the dynamics in which the system tries to seek the right balance between the energy in the field and that in the background as the field rolls down the flattening potential, before transiting in the long term to its inflationary attractor at the larger field values. One can again understand this by comparison with the case of the simple exponential: As λ_{eff} decreases the corresponding exponential attractor has an increasing fraction of scalar field energy, once the value $\lambda_{eff} < \sqrt{3}$ is reached the only solution available is the pure scalar field dominated one which becomes the late-time inflationary one if the potential has $\lambda_{eff} \sim 1$.

The "tracker" solutions in the inverse power law correspond to a case where this self-regulating mechanism in the transcient to the quintessence solution can reach a quasi-equilibrium. They have an almost constant ratio between $\dot{\phi}$ and $\sqrt{V}$, so that

$$\frac{V'}{3H\dot{\phi}} \approx \frac{M_P V'/V}{3\sqrt{\rho_n/V}} \tag{96}$$

The two limits between which the system adjusts are those in which this factor is small or large compared to unity; in the power law ρ_n changes just in such a way that this factor remains approximately constant as the numerator decreases. This self-adjustment mechanism (to 'intermediate' damping) not only fixes the ratio ξ, but drives ρ_n to a determinate amplitude. Thus the dependence in initial conditions is wiped out by this mechanism, and the transcient to the asymptotic behaviour will be determined. The time and manner in which one enters the scalar dominated era is therefore also independent of the initial conditions.

This argument, in more general, establishes not such a specific behaviour, but does show that there is a attractive dynamics associated with the approach to the inflationary behaviour, which (naturally) tends to wipe out any dependence on initial conditions. The argument depends only on very general assumptions, and as such should apply to a wide class of potentials, not just the inverse power-law: the crucial points are that the field be extremely steep $\lambda_{eff} >> 1$ when the potential density is that corresponding to the energy density in the early Universe, and that it then flattens at an energy scale V_o to smaller values of $\lambda_{eff} \sim 1$. It must also have $\lambda_{eff} < 1$ at the energy scale ρ_o when the field comes to dominate. The steep region allows the original energy density in scalar field to become extremely sub-dominant; the flattening then "stops" the scalar field when the potential energy scale is $\sim V_o$, and keeps it sitting there until ρ_n has decreased sufficiently that the condition Eq. (95) is violated. Then an evolution begins, with a dynamics in which $\dot{\phi}$, $V(\phi)$ and ρ_n attempt to "self-adjust" in a way which wipes tends to wipe out dependence on initial conditions, and from which the field transits to the required late time inflationary solution, when $V(\phi) \sim \rho_o$. Depending on the precise form of the potential there may be some weak dependence on initial conditions, but certainly the general tendency towards the inflationary solution of quintessence will be there irrespective of where we came from at early times. For the inverse power-law potential such dependence is essentially wiped out in the case that the "tracker" regime is attained.

We have not established whether the scale V_o is necessarily close to ρ_o i.e. whether the characteristic scale at which the potential becomes of the inflationary type is related to ρ_o. This is the "coincidence" problem. If the evolution in the transcient, after the overdamped phase, can allow V_o to decrease substantially this would indeed alleviate this problem (It is clear that the scale cannot be too far up near T_{rh}^4, as we need the steep part of the potential there). In the "tracker" solutions in inverse power-law potentials the scalar field energy behaves as $\rho_\phi \propto a^{-\frac{nm}{2+m}}$, so that the scalar field "tracks" the scaling of the dominant component. Since this is the evolution which governs the field between the over-damped regime where $V \sim V_o$, and today, when $V \sim \rho_o$, this would indeed seem to promise an improvement. The reason this does not work significantly can be seen from Eq. (96). In the "tracker" this quantity cannot be small, simply because if it were one is in the over-damped regime which we have described above (in which the field is "stuck"). Clearly then for the tracker to be followed when $\rho_n >> \rho_\phi \geq V(\phi)$, we must lie at the part of the potential where λ_{eff} is still large. We arrive only in the inflationary region just at the time when we have $\rho_n \sim \rho_\phi$ i.e. today by construction. Put another way the value V_o where the field got stuck in the period before entering the "tracker" cannot lie in the inflationary part of the potential, or alternatively, if we are in the inflationary part of the potential we cannot be in the tracker. Thus the tracker does not help particularly to alleviate the coincidence problem.

In fact for the same reason there is a problem in exploiting the "tracker" solutions to have robust independence of initial conditions. Starting with our "natural" initial conditions one finds that for $m < 5$ one never reaches them - the range in field space between the steep part and the inflationary part (at $\phi \sim mM_P$) is too short to attain the "tracker" behaviour associated with the intermediate range (or, to put it another way, we only transit to inflation via the tracker if we start from a more damped set of initial conditions than the "natural" ones i.e. $\rho_\phi << \rho_n$ at T_{rh}). For $m \geq 5$, on the other hand, the equation of state in the tracker is very close to that of matter, and can only evolve away towards its required inflationary behaviour (for quintessence)when $\rho_\phi \sim \rho_n$. To do this it must run further down the potential, which takes time. It is found numerically [44] that one has a minimum value (attained for $m = 5$, and $\Omega_\phi = 0.8$) of $w = -0.6$, which is at the limit of observationally consistent values. To circumvent this one must make some modification to the simple inverse power law, to flatten the potential at the appropriate point. In [44] this is achieved by adding for example an inverse power with lower exponent normalised so that it begins to dominate at $V(\phi) \sim \rho_o$ (making it manifestly clear that these tracker solutions are not helping much with the coincidence problem!).

An alternative modification to the inverse power law potential, which has this effect, is the potential of (88), which is argued to be well motivated [41] by corrections (from supergravity) to the the particle physics models which produce inverse power laws. This potential has a local minimum[16] and therefore, as in the case of (87)and (89) one enters a region well described by an ordinary power-law which, with appropriate choice of parameters, will have an inflationary solution at $V(\phi) \sim \rho_o$. One therefore can have make use of the "tracker" to get independence of initial conditions, and then have flatter behaviour take over at the appropriate point to give quintessence.

Before giving criteria for the absence of fine-tuning of initial conditions let us be a little bit more

[16]The potential given does not have its minimum explicitly at zero, and a constant needs to be added to guarantee that it is a sensible quintessence model. Alternatively one should be sure that the offset from the minimum is not playing any significant roll in the proposed quintessence solution i.e. that $V_{min} << V(\phi)$ is the relevant range of the potential.

precise about what we mean by the latter. For this it is useful to consider the potential (85) which we have not discussed. This potential has the feature which distinguishes it from all the others given that it is bounded above (at the scale Λ^4). This feature can be understood as another way of removing the problem with an ordinary (positive) power law potential, that it gives inflation at all energy scales larger than a given scale, leading to highly fine-tuned initial conditions. This potential does not, however, have the attractor behaviours at larger values of the potentials imposed in the potentials (87),(88) and (89) to get the independence of initial conditions. Does it behave like the steep potentials just discussed and show effective independence of initial conditions? The answer is in the negative. For, starting from the closest thing we can get to "natural" initial conditions $\frac{1}{2}\dot{\phi}^2 \sim T_{rh}^4$, and ϕ_o chosen randomly, the kinetic energy red-shifts away as $\frac{1}{2}\dot{\phi}^2 \propto 1/a^6$ until temperature

$$T \sim T_{rh} \left(\frac{\Lambda}{T_{rh}}\right)^{\frac{2}{3}} \tag{97}$$

which is well within the radiation dominated era (we certainly need $T_{rh} > T_{nucl}$ and we have $\Lambda^4 \sim \rho_o$) and therefore, the time evolution of ϕ is given by Eq.(91) above. Thus the field rolls by an amount of order $2M_P$ since T_{rh}, from some arbitrary field position. On the other hand in order to have inflation in the potential one needs $f > M_P$ so that the width of the potential is at least $2\pi M_P$. The field can therefore find itself at the maximum, where there is the desired late-time inflationary solution, or near the minimum, where there is none. For a given value of initial kinetic energy in the scalar field, where it arrives depends on which value of ϕ_o is chosen in the initial conditions. In terms of energy this means we must choose the initial conditions on the scalar field to an accuracy $\sim \Lambda^4 \sim \rho_o$ in energy, which is an enormous fine-tuning. By contrast when we say that there is no fine-tuning, as in the case previously discussed, we mean the following. Firstly we mean that, for any reasonable value of T_{rh} (i.e. most conservatively $T_{rh} > 1$MeV), we can fix the parameters in our potential to give a good quintessence solution (with some given Ω_ϕ^o and w^o today) for some specific set of "natural" initial conditions. Then further we require that [17]

$$|\frac{1}{2}\dot{\phi}_o^2 \frac{\partial \Omega_\phi^o}{\partial(\frac{1}{2}\dot{\phi}_o^2)}| \leq 1 \qquad |V_o \frac{\partial \Omega_\phi^o}{\partial V_o}| \leq 1 \qquad |\frac{1}{2}\dot{\phi}_o^2 \frac{\partial w^o}{\partial(\frac{1}{2}\dot{\phi}_o^2)}| \leq 1 \qquad |V_o \frac{\partial w^o}{\partial V_o}| \leq 1 \tag{98}$$

In the potential (85) which we have just discussed, these derivatives will on the contrary be $\sim (T_{rh}^4/\rho_o)$, while they are effectively zero for the potentials with independence of initial conditions.

Following on this discussion it is now easy to propose criteria for a potential which should be sufficient to ensure that it can give quintessence without fine-tuning of initial conditions. We suppose we have a potential $V(\phi)$ which is a continuous function such that

- **1**. $Inf[V(\phi)] = 0$.
- **2**. $Sup[V(\phi)] > T_{rh}^4$

Then further it suffices to consider the dynamics of its homogeneous modes in the FRW model *in which it is the only energy component.* If in this case it has

- **3**. No solutions in which the scale factor a increases faster than $a \propto t^{1/2}$ for $T_{reh}^4 > V(\phi) > \rho_o$ where ρ_o is the energy density of the Universe today (i.e. no solutions in which the energy density in the scalar field scales slower than radiation), and
- **4**. An inflationary solution (scale factor a increasing faster than $a \propto t$) for $V(\phi) \sim \rho_o$

then it will be a good candidate.

[17] We don't include the dividing factors w^o and Ω_ϕ^o because we assume them to be of order unity.

The coincidence problem is manifest in that the scale ρ_o around which the potential changes behaviour is the energy density of the Universe today.

These are neither sufficient nor necessary conditions, but approximate qualitative criteria. The first is simply one which we have argued in section 5.3 should be satisfied by definition in a quintessence model. The second condition ensures that the field starts far up the potential for "natural" initial conditions, and excludes cases like (85), which otherwise satisfy the criteria. Condition 3 ensures that the field rolls down towards its minimum, and never becomes dominated by scalar field for any significant period before today. For, if the scalar field were to dominate, the condition (which is a condition on the pure scalar field case) ensures that it scales faster than radiation. It also ensures that the field rolls to $V(\phi) \sim \rho_o$ by the time the energy density in the Universe is of order ρ_o, since if it lies further up the potential its energy density must dominate, which for the same reason just given cannot happen. Condition 4 is enough to ensure that we get quintessence today: if the scalar field on its own has an inflationary solution in this region of the potential, the addition of the matter can only make it more damped i.e. more inflationary. It is therefore impossible that the field run significantly passed this point before $\rho_\phi \approx \rho_n$.

These last two conditions can be stated approximately (and more usefully for practical purposes) in terms of conditions on the gradient of the potential, as

- **3′**. $\lambda_{eff} = M_P V'/V \geq 2$ for $T_{reh}^4 > V(\phi) > \rho_o$, and
- **4′**. $\lambda_{eff} < 1$ for $V(\phi) \sim \rho_o$

i.e. the potential needs to be of the 'steep-flat' form.

Stated this way it is most evident in what sense the conditions are neither necessary nor sufficient. We could make them more exact by adding various precisions, but we have chosen to give them as here to emphasise the underlying simple requirement, whose only real constraint is that which corresponds to the coincidence problem. Clearly we need really $\lambda_{eff} > 8$ around nucleosynthesis, and we can also tolerate regions in which $\lambda_{eff} < 2$ further up the potential, provided they drive periods of inflation (or, say, matter scaling) which are not too extended. The one potential in our list above which we have not discussed, that of Eq. (90), exploits in fact this kind of possibility in an interesting way, and in fact it is the only potential, besides (85), which does not meet the minimal form of our criteria. Rewritten as an exponential with the form of (83), the oscillatory prefactor effectively introduces a modulation in the exponent in the simple exponential (i.e. λ_{eff}). The value λ about which one modulates is taken to be in the range in which there is the attractor to the dominant component, so that the cosmology in the limit $\nu = 0$ is the "self-tuning" one discussed in section 5.2. To work as quintessence the effective exponent must decrease to inflationary values when the potential energy is around ρ_o (thus satisfying our condition 1). However its variation may also be such as to produce short periods of inflation (and scalar field dominance) at earlier epochs. So long at such periods do not occur around nucleosynthesis, nor in the era of structure formation, they will be consistent observationally. Thus the potential actually violates the condition 3, which would need to be restated in a more precise manner (to place a constraint on the duration of inflation in the various ranges of potential energy). Because of this property the potential is thus arguably less beset by the coincidence problem, as it is not just today that the potential drives a period of accelerated scalar field dominance, rather the history of the Universe can be punctuated by such phases. Note that such phases must interpolate back to the "self-tuning" attractor, so that intermittent phases of kination are also a feature of such models.

For monotonic potentials the condition that λ_{eff} decrease as V decreases can be written as the requirement

$$\frac{V''V}{(V')^2} < 1 \tag{99}$$

if $V' < 0$, or the opposite if $V' > 0$. In [44] such a condition is one of those required for what the authors call "tracking" behaviour. Here we do not add the extra conditions they give (and in general do not require the monoticity of the potential), but require only in addition that λ_{eff} reach inflationary values when $V(\phi) \sim \rho_o$.

It is easy to see then the simple power-law potentials of ordinary inflation are excluded by the general criteria we have given. Indeed for $V(\phi) \propto \phi^n$ (with n positive) the potential is inflationary (slow-roll) for $\phi > nM_P$, and therefore at all values of the potential greater than some value given by the overall normalisation. Thus condition 3 is not satisfied. On the other hand all the other potentials in our list which had independence of initial conditions do fit the criteria. To draw attention to the point we have made that the class is broader than these, let us give a few examples of cases which have neither exponential attractors nor the "tracker" solutions of inverse power-laws.

Essentially our criteria say that any generic potential steeper than a simple exponential will do in the region of the potential where we want to avoid inflationary solutions. For example we could modify the simple inflationary exponential other than envisaged by the examples we have given, taking

$$M_P^4 \exp\left[-\alpha^2(\frac{\phi}{M_P})^2\right] + \Lambda^4 \exp\left[-\beta(\frac{\phi}{M_P})\right] \qquad \phi > 0 \tag{100}$$

with Λ chosen so that the second term becomes dominant when it is of order ρ_o,which gives

$$\Lambda \approx (\rho_o)^{1/4} \times 10^{\frac{2\beta}{\alpha}} \tag{101}$$

Given that we need $\beta \leq 1$, we could have Λ orders of magnitude above ρ_o for small values of α, which is only constrained by the requirement that the first potential not be inflationary at the time of reheating, which gives $(1/\alpha) < \ln(M_P/T_{rh})$. For $T_{rh} \sim 10^{15}$GeV one could have Λ near the TeV scale. This potential will not require fine-tuning of initial conditions to give quintessence today, despite the fact that there is not an attractor or "tracker" solution at work.

In a similar spirit one could modify the inverse power law with a very steep exponential. We note in fact that the modification given by (88) is in fact just this. Instead of using initial conditions with $\phi << M_P$, as envisaged in [41] in order to pass through the "tracker" solution, one could start at the other side at $\phi >> M_P$. The system will then never pass through the "tracker", but will give good non fine-tuned quintessence solutions. An even simpler case which satisfies our criteria is the simple power-law itself, but we do not require that it reaches the tracker solution. Indeed, as we discussed, for $m < 5$ (and in particular for the interesting case of $m = 4$ which gives $M \sim$TeV) one never gets to the "tracker" starting from "natural" initial conditions. However this is not a problem as the tracker is not necessary in order to avoid fine-tuning. Indeed a recent numerical study [45] of precisely this simple case shows that the initial conditions are not only not fine-tuned, but that there is a very weak sensitivity to these initial conditions (in the sense that the derivatives in Eq. (98) are much much less than unity).

8 An alternative scenario for exit from inflation

In our discussion of a model with a phase of kination before nucleosynthesis we noted that the backward extrapolation of the model to arbitrarily high temperatures had to be treated with some caution. For the case of pure kination ($\xi = 1$, $n = 6$) lasting until a temperature T_e (of equality with radiation) we noted that at $T_{eq} \sim \sqrt{M_P T_e}$ the notion of temperature as a thermodynamic one breaks down. At the same scale the number density per horizon volume becomes of order one, which indicates that quantum effects may already become important at this scale rather than at the higher temperature T_{lim} at which the energy density in the scalar field becomes of order the Planck density. Indeed, as we will now discuss, a semi-classical treatment of quantum fields in a time-dependent background is well known to lead to particle creation from the vacuum, and the characteristic energy density associated with such effects is naturally $\sim H^4$. Thus above the temperature T_{eq} these effects would overwhelm the radiation fluid in the purely classical description, making the backwards extrapolation of this model invalid. To use such a description for the whole kination phase we must be sure that this phase starts well below $H \sim T_{eq}$, or in more general that $H << T$ in the whole range in which we apply the model.

For example consider again the model we presented in section 6.2 which gave a phase of kination beginning at temperature T_o starting from "natural" initial conditions after a phase of ordinary inflation leaving the Universe at temperature T_{rh}. The scalar field had a two stage evolution, first coming to dominate in a phase in which the scalar field energy density $\rho_\phi \propto a^{-\lambda_1^2}$ (and $\lambda_1 < 2$), and then red-shifting away in a kination phase ending at T_e. For this model to be valid (in the sense we have just discussed, that the quantum mechnical particle creation effects are negligible) we need to check not just that $T_o << T_{eq}$, but in general that throughout the evolution $H << T$. It is true at T_{rh} simply because $T_{rh} << M_P$; in the subsequent phase of scalar field domination

$$\frac{H}{T} \approx \frac{T_{rh}}{M_P} \left(\frac{T}{T_{rh}}\right)^{\frac{\lambda_1^2}{2}-1} \tag{102}$$

so that the constraint is trivially satisfied unless $\lambda_1^2 < 2$, and in that case it will put a bound on the duration of the phase of inflation for the validity of the model. Further in the kination phase this is of course a decreasing function. Thus, given that, for a kination phase at the electroweak scale we saw that we required only $\lambda_1^2 \leq 3$, the model is quite consistent, and the effects of particle creation are small corrections.

8.1 Gravitational reheating with a phase of kination

The fact that H/T increases in inflation corresponds to the appearance of the horizon in this case: the "classical" radiation is red-shifted out of the horizon and after a finite time number of e-foldings the quantum-particle creation, with associated "temperature" $T \sim H$ dominates. Contrary to what is often said the Universe is not "cold and empty" after inflation, but rather has this associated temperature, like the Hawking temperature of a black hole. It was first noticed by Ford [46] that this could in principle be enough to give the required radiation dominated FRW phase after inflation: If instead of decaying (as in standard reheating scenarios) the scalar field rolls into a mode scaling faster than radiation[18] i.e. by interpolating with a period of what we called kination one can get from the inflationary phase to the ordinary radiation dominated FRW phase in a way which involves only gravity, rather than by the decay of the inflaton through itds interactions as envisaged in the standard reheating scenario. Indeed we have assumed here that the scalar field is appropriately weakly coupled, which as stressed in [47] places extremely stringent bounds on such couplings. We will return briefly to this point when we discuss all these models from the point of view of Particle Physics.

Let us be a little more precise. We suppose matter fields minimally coupled to gravity. In this case there is particle creation only for the non-conformally coupled[19] scalar fields, while the gauge and fermionic fields are not sourced (because of the conformal invariance of their actions). From a calculation [46, 48] in which the de Sitter phase is matched smoothly onto the FRW phase it can be shown that the energy

[18] Ford explicitly works the example of a scalar field oscillating about the minimum of a polynomial ϕ^n with $n > 4$, as discussed in section 3.

[19] A scalar field is conformally coupled when it has a coupling $\xi \phi R$ to the curvature, with $\xi = 1/6$. Particle creation effects are proportional to $(\xi - 1/6)$, and are in particular non-zero for our minimal coupling of $\xi = 0$.

density per light ($m << H$) scalar degree of freedom in the FRW phase is given by

$$\rho_{rad}(a) = \epsilon H_I^4 \left(\frac{a_I}{a}\right)^4 \tag{103}$$

where H_I is the expansion rate in the de Sitter phase ending when the scale factor is a_I, and $\epsilon \approx 10^{-2}$, which up to this latter numerical factor is just what we have been assuming in our naive estimates above. We suppose that we have N_s light scalar degrees of freedom, which are strongly coupled to the other fermionic and gauge degrees of freedom e.g. the Higgs doublet of the standard model, or the many scalar degrees of freedom in standard supersymmetric extensions like the MSSM. Assuming they have a spectrum peaked around wavenumber $H_I a_I/a$, they will equilibrate with one another and/or with the other degrees of freedom when the corresponding rates $\sim \alpha H_I a_I/a$ (where α parametrises the strength of the couplings) becomes of order the expansion rate i.e. for pure kination ($n = 6$) when $a_I/a \sim \sqrt{\alpha}$. If at this time the equilibration occurs between g_*^i degrees of freedom the thermodynamic temperature at this time is then given by

$$T_i \approx \sqrt{\alpha}\left(\frac{30\epsilon N_s}{\pi^2 g_* i}\right)^{1/4} H_I \tag{104}$$

since $\rho_{rad} = \pi^2 g_* T^4/30$. From this temperature on the model is well described by the "classical" one with a radiation fluid at temperature $T \propto 1/a$ dominated by the scalar field red-shifting away its energy as $1/a^6$. The requirement that the transition to radiation domination occurs before nucleosynthesis and in keeping with it, gives us a lower bound on the parameter H_I. For the case of pure kination (with $n = 6$) we have that this transition takes place at T_e given by[20]

$$T_e \approx \left(\frac{\epsilon N_s}{3}\right)^{1/2}\left(\frac{30\epsilon N_s}{\pi^2 g_* i}\right)^{1/4} \frac{H_I^2}{M_P} \tag{105}$$

where we neglect the effect of any adiabatic decouplings of degrees of freedom between the temperatures T_i and T_e. The condition that the minimum value consistent with nucleosynthesis is $T_e \sim 5$MeV, which corresponds to a lower bound of H_I in the range $10^7 - 10^8$GeV. The lower end of the range will give kination ending just before nucleosynthesis, while inflation at the GUT scale gives $H_I \sim M_{GUT}^2/M_P$ and therefore $T_e \sim (M_{GUT}/M_P)^4 M_P \sim 10^6$GeV. Note however that we have assumed a sudden change from pure de Sitter to pure kination; the effect of a smooth interpolation can have inflation at the GUT scale with the temperature T_e as low as the MeV scale [26].

In [26] we have constructed an explicit model of this kind giving kination at the electroweak scale, sufficiently simple to permit an analytical calculation. Again it is a two step exponential exactly like that in Eq.(71), with first a part flat enough to drive inflation and subsequently a steep exponential giving pure kination. There are thus three model parameters - the two exponents (required to be in the range for inflation and kination respectively) and the characteristic mass scale at which the potential changes slope. Fixing the temperature T_e for the transition to radiation domination gives one constraint; one can then also calculate the amplitude and exponent of density perturbations generated during inflation. If we require that (i) the kination phase be pure kination ($n = 6$) ending at T_e close to its lower limit ($\sim$MeV), so as to produce the dramatic effects on electroweak physics we discussed briefly in section 10, and (ii) that the amplitude of the perturbations is in agreement with that required by the COBE measurements, we find that in this model there is a non-trivial prediction of the exponent in the spectrum $P(k) \propto k^p$, with $p \approx 0.7$.

[20] Up to the prefactors this of course just gives H_I as the scale we previously called T_{eq} corresponding to the maximum temperature for backward extrapolation of the simple classical cosmology with a phase of kination.

This model, despite being simplified to allow analytical calculation, shows that in this kind of model there will be non-trivial constraints linking density perturbations to the post-inflationary cosmology. For a realization of the scenario with an oscillating mode we found, on the contrary, that such constraints could not be satisfied: the production of density perturbations requires a self-coupling of the scalar field which leads to an extremely efficient decay of the condensate field in its oscillating mode through non-linear "parametric resonance" effects.

8.2 A variant with both kination and quintessence: "Quintessential Inflation"

An evident extension of models of this type is one in which the same scalar field ultimately comes back to play a role at late times, or in particular as quintessence. It was Spokoiny[49] who proposed a model of gravitational reheating[21] with precisely the goal of identifying the inflaton field and (what is now known as) the quintessence field. This is a scenario which can clearly be implemented with a flat-steep-flat form for the potential e.g. in our toy model with the addition of a third piece to the exponential with an inflationary exponent. A somewhat less contrived example would be the potential we wrote down above in Eqs. (100), which, for an appropriately small α can give inflation, then kination, and finally quintessence today.

Peebles and Vilenkin[50] have studied this particular scenario, which they call "quintessential inflation", with a toy potential

$$\begin{aligned} V(\phi) &= \lambda(\phi^4 + M^4) \qquad \phi < 0 \\ &= \lambda\frac{M^8}{\phi^4 + M^4} \qquad \phi < 0 \end{aligned} \tag{106}$$

which, being a standard chaotic inflation potential at $\phi << -M$, and an inverse power law quintessence potential at $\phi >> M$, with a steep $(M_P V'/V >> 1)$ region interpolating, is of the required flat-steep-flat type. Fixing λ to give the density perturbations required by COBE, and M to give late time quintessence, the temperature T_e is determined, and is estimated by Peebles and Vilenkin to be $T_e \sim 10^3 N_s^{3/4}$ GeV (where N_s is again the number of light scalars sourced by gravitational reheating). Thus in this case the kination phase terminates before the electroweak scale and the effects we have discussed there do not take place. It is noteworthy however that the scales are not too far apart, and in a smoothened form of this potential, leading generally to a longer interpolating phase, it may well be that T_e could be low enough to modify electroweak physics. It is interesting to note here again, as in the model analysed in [26], the requirement that the model produce also acceptable density perturbations allows the model to become potentially more predictive.

Peebles and Vilenkin note one important observational constraint on this kind of model (with or without the late time quintessence) overlooked in the previous work: gravitons are also efficiently produced by the same mechanism producing the scalar particles which generate the radiation of the hot radiation dominated FRW Universe. One therefore expects very copious production of gravitational waves to be associated with this scenario in comparison to the standard scenario in which a radiation dominated phase follows inflation. The most direct constraint is not from gravitational waves detectors, which do not have much sensitivity to the corresponding very short wavelengths (characterised by the horizon scale during kination), but from the direct effect on nucleosynthesis requiring the energy density associated with the stochastic gravitational wave background to be no more than that permitted at that time. The gravitons are equivalent to two scalar degrees of freedom, so that the fraction of the energy density associated with them will be approximately $2/(2+N_s)$. To have this less than ~ 0.05 requires $N_s > 40$ i.e. at least forty scalar degrees of freedom with mass $m << H_I$, the expansion rate at the end of inflation. While in the MSM we have only the single

[21] Spokoiny was apparently unaware of the earlier work of Ford, as we were at the time of writing of [4, 26, 32, 33].

Higgs doublet of complex scalar fields, and therefore $N_s = 4$, in almost any extension of this model one will have large numbers of these scalar modes. In any supersymmetric model there will be all the superpartners of the fermions, which typically will have mass scales around the SUSY scale which means they will always be light in the sense required here (recall we found that the lower bound on H_I is of order 10^7 Gev). In the MSSM for example $N_s = 104$, so that the nucleosynthesis constraint is comfortably satisfied.

It remains that the stochastic background of gravitational waves at the corresponding wavenumbers will have significant power in a range where it is negligible in standard models of reheating. This would seem to offer one hope of a direct probe of such a phase. A detailed study has been carried out in [17], where the author finds that the amplification in the corresponding range of high frequencies (∼GHz) can be as large as eight orders of magnitude with respect to the standard result. While current interferometer experiments are not sensitive to this range, it is plausible that future measurments with microwave cavities could detect a signal at this amplitude.

9 Scalar fields in cosmology and Particle Physics

The approach of this review has been to start from cosmology, examining in quite a general way the possible effect of any scalar field in a homogeneous mode to the standard FRW type cosmology, without assuming any a priori constraints on what the scalar field potential, or indeed what the origin of this field is. When we built models the only criterion, albeit unstated (and perhaps sometimes unfulfilled), was that we sought a simple functional forms for the potentials. This was not difficult once we had identified the particular properties of a few specific potentials.

We now discuss briefly the Particle Physics of the models we have been discussing, drawing attention to some of the problems posed to model-building by such scenarios. We divide our discussion into two parts: a discussion of the basic assumption we have made everywhere that the scalar field was completely uncoupled from other matter, and a discussion of the mass scales involved in the potentials needed. We look at the second first, as it is crucial to the question of couplings. We do not enter in any detail the question of model building in specific models. A review of attempts in this direction, in particular in the context of supersymmetric theories beyond the standard model, can be found in [51].

9.1 Mass scales

Whenever we wrote explicitly the time variation of the condensate fields in our examples we have seen that the characteristic variation of the condensate field over cosmological time is

$$\Delta\phi \sim M_P. \tag{107}$$

The reason for this is simple to see: If we have a field that is rolling while contributing significantly to the energy density of the Universe, this means that its kinetic energy is a non-negligible fraction of the total energy, or, roughly

$$\dot{\phi} \sim \sqrt{\rho_{tot}} \sim HM_P \tag{108}$$

Now if further the damping term is playing an important role in the dynamics (i.e. $H\dot{\phi} \sim V', \ddot{\phi}$) the time-scale for the variation of ϕ is given by H^{-1} and the result (107) follows. Therefore, in all these scenarios with a rolling scalar field, we are considering condensates at the Planck scale, and therefore inevitably the Particle Physics context is those theories which seek to unify gravity and quantum mechanics e.g. string theory.

The exception is the case of a field which oscillates about a minimum, in which case the damping term is very sub-dominant and the time-scale for the variation of ϕ is set by the frequency of oscillation ω, which depends on the parameters in the potential and not on the cosmology (for a quadratic potential $\omega \sim m$). For analytic potentials such a behaviour can give rise only to a scaling faster than matter i.e. $\rho_\phi \propto 1/a^n$ with $n \geq 3$, and so we have only exploited such a behaviour in one of our models of kination. The amplitude of the variation of the field is then

$$\Delta\phi \sim \frac{H}{m_{eff}} M_P \qquad m_{eff}^2 \sim V''(\phi) \tag{109}$$

which is now valid for $m_{eff} >> H$ (the oscillation begins at $m_{eff} \sim H$). Thus, for kination at the electroweak scale as we envisaged in the oscillating model in section 6.2, it suffices to have a polynomial of a high power with mass scale $m_{eff} >> M_{ew}^2/M_P \sim 10^{-5}$eV. For a weak scale mass the variation of the condensate will be of order the weak scale, and certainly the potential may have no relation at all to Planck scale physics. One possible problem with this sort of oscillating model compared with damped rolling models is presented by resonance effects which can occur, leading to a very efficient decay of the condensate. Neglecting the feed-back from perturbations in dark matter the linearized equation in k-space for inhomogeneous perturbations ψ to the condensate is given by (see [33])

$$\ddot{\psi} + 2\mathcal{H}\dot{\psi} + (k^2 + a^2 V''(\phi))\psi = 0 \tag{110}$$

where the notation is as in Eq.(47) above. When the field ϕ is oscillating in a non-linear potential $V(\phi)$ there will generically be resonances (just like those of a driven oscillator in classical mechanics - this is so-called "parametric resonance"). The perturbations in certain channels are enormously amplified, corresponding quantum mechically to particle creation and the decay of the condensate. This effect has been much studied in the context of standard inflation as a mechanism for efficient reheating of the Universe [52]. In [26] we have studied these constraints numerically for the case of kination in a ϕ^n potential before nucleosynthesis. While it is found to be possible to have a condensate long-lived enough, the constraint on simple dimensionless couplings g to other scalar fields is found to as restrictive as $g < 10^{-20}$.

The formula in Eq. (109), applied again to the damped roll case, gives $m_{eff} \sim H$. This is actually true because we have that $m_{eff}^2 \sim V''(\phi) \sim V/M_P^2 \sim H^2$. This is the second major constraint we have on any damped roll model: *the mass scale associated with the scalar field is of order the expansion rate at the time it plays a role.* For kination lasting to near the nucleosynthesis scale this this means that at most $m_{eff} \sim 10^{-15}$eV, while for both the "self-tuning" field and quintessence it means that $m_{eff} \leq 10^{-33}$eV. Note that this does not mean that such a scale necessarily appears explicitly in the potential - we have seen that it does not for either the exponential or inverse power-law. Rather it corresponds to the fact that the field has to roll far along the potential where the energy is $\sim \rho_o$. This places an extremely tight constraint on such models as it means that any corrections to the supposed potential must not generate an effective mass for this field bigger than these tiny values. Further, as we will discuss further below, these tiny masses themselves put extremely strong constraints on the couplings of these scalar fields to observable sectors, since they would lead to long range forces generically violating the Equivalence Principle. It is these stringent requirements of the particle physics of these models which make model building so problematic (see [51] and references therein).

We have alse seen that besides M_P at least one very distinct mass scale M must be introduced in almost all models we have considered, related to the required change in behaviour of the potential at some characteristic scale. In quintessence this is the "coincidence" problem: this scale must be chosen to be just such that the scalar field returns to dominance today. While in simple models with exponentials, this scale is almost directly related to $\rho^{1/4} \sim 10^{-3}$eV, in inverse power law potentials or in a potential like that in Eq. (100) it can be at a much higher scale, above the EW scale, which from the point of view of particle physics model building makes it much more plausible. As discussed in [38] such potentials can arise for example in the description of SUSY breaking at the scale M.

9.2 Couplings to visible sectors

When the supposition which we have made that the scalar field is uncoupled to other sectors (other than through its minimal coupling to gravity) is relaxed, we can divide the implications to be considered into two parts:

- The modifications which result to the cosmological evolution of the condensate;
- The observational constraints in general on such couplings.

For what concerns the first the effect of introducing couplings is in general extremely non-trivial. When we wrote down the classical equation for the condensate, we wrote in fact an equation for the one-point function of the quantum field ϕ, and hid all the complexity behind this by prescribing $V(\phi)$ to be the effective potential. We can generalise this simple approach to more than one field if we wish only to describe condensate states in all such fields. Generically however we will want to describe the effect of couplings to fields which are not restricted to have all their energy in a homogeneous condensate. An evident case is that of couplings leading to the perturbative decay of the inflaton into other lighter degrees of freedom, which can be described with certain assumptions by the addition of a term $\Gamma\dot{\phi}$ to the equation of motion we have treated, where Γ is a perturbative decay rate. Clearly when $\Gamma \sim H$ i.e. when the life-time of the inflaton particles becomes of order the age of the Universe the equation for the condensate has an exponentially decaying solution, and it simply disappears. Clearly for the models we have described we would need at the very least that the effective rate Γ induced to other couplings is less than H at all times for which the condensate has been assumed to play a role i.e. for quintessence we will need the ϕ particles to have an extremely weak coupling in any channel allowing these particles to decay.

Given that the field are in any case constrained to have such tiny effective masses as they roll further down the potential, this condition is not highly restrictive in these models, as it essentially means one needs only to avoid couplings which would allow decays into less massive particles (which, given that the mass scale 10^{-33}eV is the horizon scale, does not even necessarily include photons!). This tiny mass imposes much tighter constraints through the second consideration above: Such a scalar field, if coupled to any ordinary matter, will induce long range forces between particles which generically violate the Equivalence Principle, on which there are strong constraints through Eotvos type experiments. Given a real scalar field as we have assumed, it must of course be a singlet of the standard model gauge group. There are therefore no normalisable terms which can be written down coupling the field to standard model fields. In general however, assuming that there is some underlying fundamental theory (e.g. string theory) which is finite, with an intrinsic UV cut-off at some scale M we expect there to be an infinite number of higher dimensional (non-renormalisable) operators coupling ϕ to the standard model fields e.g.

$$(\frac{\phi}{M})^n \mathcal{L}_m \tag{111}$$

where $\mathcal{L}_m$ is the renormalisable standard model Lagrangian, and $n = 1, 2, ...$ Some specific bounds which have derived are given in [53]. Couplings to the QCD gauge fields G of the form $\beta(\phi/M)Tr(G^2)$ are conservatively bounded as

$$\beta \leq 10^{-4} \frac{M}{M_P} \tag{112}$$

so that even if the scale M is put at the Planck scale the bounds are tight. In the context of the Particle Physics model building which may give rise to such potentials it is a considerable problem to understand why such terms should be so suppressed. One way out - the imposition of symmetries to forbid the dangerous terms - is frustrated by the fact that the condensate itself must spontaneously break the symmetry. The association of the extreme, but not exact, flatness of the potential with an approximate symmetry (i.e. approximate Golstone boson) may be one promising way of reconciling the small mass and small couplings.

10 Some variants on simple scalar field models

We finally mention some modifications of the simple framework we have worked in throughout this review, of a scalar field with a canonical form of its action coupled minimally to gravity, and completely decoupled from other sectors. These comments bear on quintessence, as to our knowledge such extensions have not been considered for the other models we have described.

One framework in which there are necessarily scalar fields non-minimally coupled to gravity is Brans Dicke theory, or generically scalar-tensor theories of gravity. It has been shown that the scalar field can, for appropriate functional dependences in the non-minimal coupling, play the role of the quintessence field, consistently with the observations constraining the deviations from General Relativity in these theories [54]. The attractor properties we have discussed for various potentials carry across, and indeed, as emphasised in [55], generalise in certain cases to be part of a more general dynamical attractor bahaviour of these theories towards General Relativity. Corresponding to the non-trivial additional coupling there are modifications of the evolution, and possible associated observable signatures.

Several other variants are those which attempt more directly to solve the coincidence problem. The essential idea is that the latter would be resolved, or at least strongly mitigated, if the onset of the inflationary behaviour of the potential could be tied dynamically to another cosmological event not so far from the present: the transition to matter domination. In this context [56] couplings to dark matter have been proposed, argued to be well motivated in string theory, which produce a coupling between the equation describing the dark matter and the scalar field. The additional term in the equation of motion of the scalar field depends on the fractional contribution of the energy density in the dark matter, and thus plays an important role in recent epochs, leading to an effective damping of the scalar field which slows down its scaling. One can devise models which use this effect to transform an early time attractor behaviour like that we have in a simple exponential potential into a late time attractor giving acceleration, which for an appropriate value (small, but not tiny) of the relevant coupling gives quintessence today. Another type of model "k-essence" [57] involves a modification to the canonical kinetic terms in the scalar field Lagrangian, and again makes onset of matter dominance important in the onset of the accelerating behaviour. While such models are interesting they are often rather ad hoc and ultimately the coincidence problem is transferred to the fixing of the value of some extra effective coupling which is itself unobservable.

11 Conclusion

It is easy to get lost in the construction of various scenarios with scalar fields and to almost forget the enormous difficulties involved in finding any observational support for such models. Let us return finally to consider what the possible sources of such constraint are, and to ask how realistic it is that we will observe anything which may direct us specifically to this explanation of the phenomena we are trying to explain.

(i) Supernovae observations have been at the source of much of the interest in the role of scalar fields in the particular guise of so-called quintessence. Current Sn data, taken alone, allow one to say little more than that the Universe has recently entered an accelerating phase, while leaving open a quite wide range of (Ω_ϕ, w). In particular there is nothing that favours quintessence over a simple cosmological constant. A very ambitious satellite based program of Sn observation (SNAP) is now being planned, and studies show [58] that the projected statistics would be sufficient to be able to distinguish between a whole range of models from a cosmological constant to many proposed quintessence models.

(ii) With the success in recent years in observations of fluctuations of the CMBR, with results stunningly consistent with the predictions made by standard cosmological models, the idea that one will be able to derive extremely accurate information about the parameters of our current models from future more accurate observations is widely accepted. This is certainly going to continue to be one of the main ways in which constraints will be placed on the history of the Universe since a red-shift of about 10^6. If all continues with future observations in such consistency with standard models, this will certainly be the source of very tight constraints on the possible contribution from a scalar field.

(iii) Even if these cosmological observations turn out to support very strongly a very specific modification of the standard FRW model with a scalar component along the lines we have described, there would remain a theoretical leap in assigning the observed behaviour to this origin: the component is dark (and essentially homogeneous, although there are some speculations to the contrary [59]) and what we measure is essentially an effective (possibly time varying) equation of state. That it has this specific fundamental origin would be very difficult to determine. In this context certainly the discovery of scalar particles at terrestrial experiments - even if of course these scalars could not possibly be the dark cosmological ones - would add an important plausibility to the hypothesis.

(iv) For what concerns constraint on how the energy density in the Universe evolves before nucleosynthesis, we discussed the possibility of linking the baryon asymmetry and the dark matter abundance, in a framework like nucleosynthesis in which the expansion rate is in principle the only unknown variable. Obviously for this to have any plausibility as a description of the early Universe will require at least one (and ideally two) enormous experimental leaps forward: the discovery of a symmetry breaking scalar sector beyond the MSM with the right structure (including considerable CP violation) to make baryon production possible, and/or the discovery of a WIMP candidate with a coupling which makes it decouple before nucleosynthesis.

(v) Finally we mentioned one very interesting possibility, discussed quantitatively in [17], of the detection of the characteristic stochastic gravitational wave background at very high ($\sim$ GHz) frequencies associated with a phase of kination before nucleosynthesis. It would be interesting to look further at the feasibility of detecting such a background, which, if discovered, would give a direct probe of an epochs in the early Universe which are currently far beyond our reach.

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