# NOTES & CORRESPONDENCE

## FRESNEL DRAG AND THE PRINCIPLE OF RELATIVITY

**By Ronald Newburgh***

### 1. Fresnel drag

One of the most influential concepts for the development of physics in the nineteenth century was Augustin Fresnel's drag theory concerned with entrainment of the ether by moving transparent bodies.[1] A theoretical idea advanced in 1810 to explain the lack of sensitivity of stellar aberration to the direction of starlight, it was seemingly confirmed experimentally in 1851 by Hippolyte Fizeau, who measured the velocity of light in moving water. In this note I wish to analyze the generalization of Fresnel's formula to a principle which asserts the impossibility of observing first-order effects arising from motion of the earth through the putative ether. This generalization was based on much experimental work by Éleuthère Mascart and theoretical analysis by Alfred Potier and W. Veltmann. The discovery of this principle led to the awareness of the need for second-order experimentation such as the Michelson-Morley experiment. By discussing Fresnel drag in terms of the special theory of relativity, I also wish to show how the Potier-Veltmann principle foreshadows the work of Einstein.

Consider a transparent body such as a block of glass which contains ether. How is this ether affected by motion of the glass? By assuming an elastic ether Fresnel derived an expression for the velocity of the ether in the glass when the glass moves through the ether with velocity $v$. Let $K$ be the absolute frame in which the ether extraneous to matter is at rest. Let $K'$ be the frame in which a block of glass is at rest, such that $K'$ moves with velocity $v$ with respect to $K$. Fresnel showed that the ether-drift velocity in the glass as measured in $K'$ is $-v/n^2$, where $n$ is the index of refraction of the glass. Now if $c/n$ is the velocity of light in glass at rest with respect to the ether, the velocity of light in the glass in frame $K'$ is

$$c'_g = c/n - v/n^2 = c/n + (1 - 1/n^2)v - v \tag{1}$$

By applying a Galilean velocity transformation we find the velocity of light in glass as measured in frame $K$ to be

$$c_g = c/n + (1 - 1/n^2)v \tag{2}$$

The factor $(1 - 1/n^2)$ is the Fresnel drag coefficient. In short, the ether in a transparent body is entrained with velocity $(1 - 1/n^2)v$ when the body itself moves with velocity $v$ with respect to the absolute ether frame.

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[1] Augustin Sesmat, *Systèmes de référence et mouvements* (Paris: Hermann, 1937), Vol. I, Cah. 6, "L'optique des corps en mouvement," pp. 494–499. Edmund Whittaker, *A History of the Theories of Aether and Electricity* (London: Nelson, 1958), Vol. I, pp. 109–111.

### 2. A classification of nineteenth-century experimentation with moving bodies

Any experiment may be looked on as consisting of a light source $S$, a detector $D$, and an optical black box $OBB$ which operates on the light. This black box contains optical elements such as lenses, mirrors, or interferometers and may be evacuated or not. A Type I experiment is one in which there is some relative motion between two of these three elements $S$, $D$, and $OBB$. Consider stellar aberration. The source is a star and the detector is an earth-bound telescope. The telescope is also our optical black box. The star and telescope move with respect to each other so that relative motion occurs. Another Type I experiment is that which demonstrates the Doppler effect. Here, too, source and detector are in motion with respect to each other, and a measurable effect is found. Both stellar aberration and Doppler shift are effects proportional to $v/c$ where $v$ is the relative velocity between source and detector. In the nineteenth century this velocity $v$ was often interpreted in terms of the velocity of the ether wind.

In addition to Type I experiments there were many others which were designed to demonstrate and measure the ether wind. In these experiments the source, detector, and optical black box were all at rest in the laboratory. No relative motion occurred. This type of experiment we designate as Type II. A fine example of a Type II experiment is that of A. A. Michelson and E. Morley. The source, detector, and interferometer were all at rest in the laboratory. It was truly an experiment without moving parts.

However, to a nineteenth-century physicist motion was present in a Type II experiment; moreover, the existence of this motion was self-evident to him. Since the velocity of light propagation in the stationary ether was $c$ (or $c/n$ if the propagation took place in a medium at rest with respect to the ether), the presence of an ether wind of velocity $v$ with respect to the apparatus meant the necessary vectorial addition of this velocity $v$ to that of the light in a stationary ether. Before Einstein's 1905 paper almost no one doubted that the Galilean velocity addition law was valid.[2] The Type II experiments were designed on the theoretical basis of an additional velocity component $v$, provided by nature herself. They sought to measure the velocity of the ether wind at the earth's surface.

### 3. Experimental measurement of Fresnel drag

Fresnel's derivation of the drag law was theoretical, but the results were consistent with the Arago experiment in which a prism was placed in front of a telescope and the stellar aberration measured as a function of the celestial direction of the starlight. Two experiments, one each of Type I and Type II, confirmed the Fresnel drag theory. The first of these was Fizeau's Type I experiment which is sketched in Figure 1. A partially reflecting mirror $M_1$ reflected light from a source $S$. A lens $L_1$ made the light parallel, after which it was separated into two beams by the slits $A$ and $B$. Each beam passed through the tube $T$ filled with flowing water. At the mirror $M_2$ each beam was deflected so that it would return through the tube. Beam $A$ returned through slit $B$ and beam $B$ returned through slit $A$. The beams passed through $M_1$ and were combined at $S'$ where the fringe pattern was observed. When the water flowed, a fringe shift occurred.

To analyze this experiment let $\alpha$ be the Fresnel drag coefficient. Beam $B$ propagates

[2] Henri Poincaré gave a lecture in 1904 in which he asserted that there must exist a new dynamics characterized by the rule that no velocity can exceed the speed of light. See Whittaker, *History*, Vol. II, pp. 30, 31 and 37, 38. He developed the idea further in his famous paper "Sur la dynamique de l'électron," which appeared in the *Rendiconti del Circolo matematico di Palermo*, 1906, *21*:129–176. This paper has been republished in *Oeuvres de Henri Poincaré* (Paris: Gauthier-Villars, 1954), Vol. IX, p. 494. The new velocity addition rule appears on p. 500.

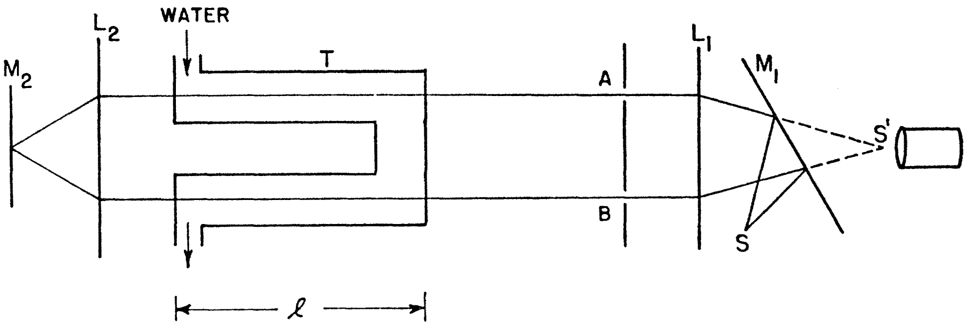


***Figure 1. Fizeau experiment.*** A source of light at *S* is reflected by the semi-reflecting mirror $M_1$ and made parallel by the lens $L_1$. The wave front is divided by slits *A* and *B*. The two beams pass twice through the tube *T* filled with flowing water. The mirror $M_2$ reverses the beam directions. A fringe pattern is observed at $S'$.

in the direction of water flow for both legs of the path and therefore propagates with velocity $c/n + \alpha v$. Beam *A* propagates against the water flow and therefore with velocity $c/n - \alpha v$. If $l$ is the tube length, the time difference for the two beams in going from *S* to $S'$ due to the water's motion is

$$\Delta t = 2l/(c/n - \alpha v) - 2l/(c/n + \alpha v) \tag{3}$$

This $\Delta t$ is in addition to that which would occur when the water is at rest. Fizeau measured an observable shift for a water velocity of 7 m/sec.[3] This gave an $\alpha$ smaller than unity. Later experiments confirmed that $\alpha$ equalled $1 - 1/n^2$, consistent with Fresnel's prediction.

Mascart, using a Jamin interferometer, carried out a Type II experiment which was a more careful repetition of an experiment by Martin Hoek and was similar in many ways to Fizeau's Type I experiment.[4] As shown in Figure 2 light from source *S* was directed to a prism $P_2$ by a prism $P_1$. Part of the light was sent in air to prism $P_3$, reflected internally within $P_3$, and returned to $P_2$ through the stationary water in tube *T*. The other part of the light traversed the same path in the opposite direction. The two beams then recombined in $P_1$ and were observed by the detector *D*. All measurements were made in the frame in which the water was at rest. Therefore the ether wind outside the water was blowing with velocity $v$, presumably the velocity of the earth with respect to the ether.

Let the velocity of the wind in the tube be $\alpha v$. For beam 1 the velocity in air is $c + v$ and the velocity in water (propagating against the wind) is $c/n + \alpha v - v$. For beam 2 the velocity in air is $c - v$ and in the water (propagating with the wind) $c/n - \alpha v + v$. The times of flight for the two beams 1 and 2 are $t_1$ and $t_2$, where

$$\begin{aligned} t_1 &= l/(c + v) + l/(c/n + \alpha v - v) \\ t_2 &= l/(c - v) + l/(c/n - \alpha v + v) \end{aligned} \tag{4}$$

[3] Hippolyte Fizeau, "Sur les hypothèses relatives à l'éther lumineux," *Annales de Chimie et de Physique*, 1859, *57*:385–404.

[4] Éleuthère Mascart, "Sur les modifications qu'éprouve la lumière (2ème partie)," *Annales Scientifiques de l'École Normale Supérieure*, 1874, *3*:363–420. Martin Hoek, "Determination de la vitesse avec laquelle est entrainée une onde lumineuse traversant un milieu en mouvement," *Archives néderlandaises des Sciences Exactes et Naturelles*, 1868, *3*:180–185.

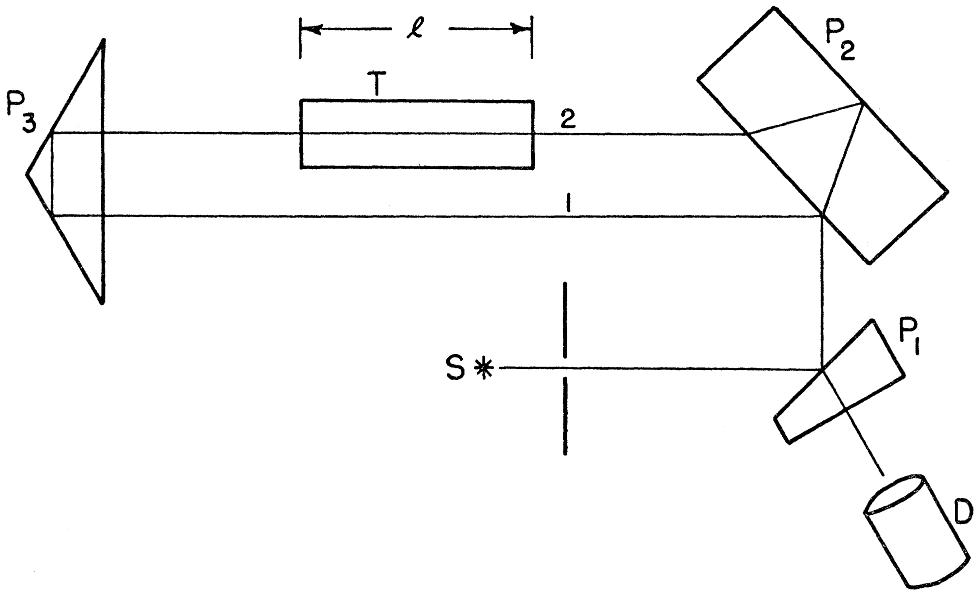


***Figure 2. Mascart-Jamin experiment.*** Light from the source $S$ is divided into two beams by the prism $P_2$. Each beam traverses the water-filled tube $T$ plus a path in air. The fringe pattern of the recombined beams is observed by the telescope $D$.

(We need not consider the portions of the path outside $l$.) Although he expected these two times to differ, Mascart never observed a fringe shift no matter how he changed the orientation of the interferometer. Since a difference between $t_1$ and $t_2$ would necessitate a fringe shift, Mascart concluded that $t_1$ equalled $t_2$ at all times. He then equated the two expressions in Equation (4). Assuming that $v$ was much smaller than $c$ and neglecting terms of the order of $v^2/c^2$, he again found $1 - 1/n^2$ as the value for $\alpha$.

#### *4. A synopsis of Mascart's papers of 1872 and 1874*

Mascart in two papers summarized all the experiments known to him which had been designed to demonstrate the motion of the earth through the ether.[5] Many of these are described in Sesmat and Whittaker,[6] but for the complete details one should read Mascart himself. In addition to the experiments done by others, Mascart described those which he had repeated plus new ones of his own design. These papers are a contemporary history of the optics of moving bodies from Arago's prism to 1874. His conclusions were startling. With one exception only, no experiment had demonstrated the motion of the earth through the ether to the first order in $v/c$. The one exception was an experiment by Fizeau in which he had observed a rotation of the azimuth of polarization for polarized light sent through a series of glass slides. This rotation Fizeau attributed to the ether wind. Mascart was unable to repeat this result, nor could any one else. It was generally accepted that an experimental flaw (perhaps a temperature gradient) had caused the result. The experiments described by Mascart

[5] Éleuthère Mascart, "Sur les modifications qu'éprouve la lumière (1ère partie)," *Ann. Sci. École Norm. Supér.*, 1872, *1*:157–214, and "Les modifications (2ème partie)."

[6] Sesmat, *Systèmes*, pp. 485–613, and Whittaker, *History*, Vol. I, pp. 94–127.

cover the following subjects: Doppler experiments, reflection from a moving mirror, diffraction, double refraction (linearly polarized light), double refraction (circularly polarized light), refraction in a prism, experiments with refraction (e.g., George B. Airy's water-filled telescope), Newton's rings, Hoek's experiment (*cf.* Mascart-Jamin). Mascart's conclusion is best given in his own words:

> . . . the translatory motion of the earth has no appreciable effect at all on the optical phenomena produced with a terrestrial source or with solar light. These phenomena are incapable of demonstrating the *absolute* motion of a body. Relative motions are the only ones we can make evident.[7]

In the terminology we have introduced we might paraphrase Mascart's conclusion by saying that all Type II experiments give null results. Demonstrable effects are observable in Type I experiments only. This is an inductive law based on generalizations from experiment.

### 5. *Potier-Veltmann principle: The impossibility of observing first-order terrestrial ether effects*

Veltmann provided the first theoretical justification for Mascart's conclusion. In a series of papers he examined the propagation of light in moving media in relation to the Fresnel drag theory.[8] He showed that Snell's law (and therefore the index of refraction) is the same for stationary or moving media, if one assumes the Fresnel theory. This is, of course, the explanation of the null result of the Arago prism experiment as well as Airy's experiment in which stellar aberration was examined with a water-filled telescope. Veltmann also showed that interference phenomena are independent of the state of motion of the medium.

Potier was aware of Veltmann's papers, which though invaluable, were somewhat inchoate. By combining Fresnel's theory with Fermat's principle of least time, Potier showed elegantly and succinctly that to the first order in $v/c$ absolute motion with respect to the ether is undetectable by optical means.[9]

The Fresnel theory states that the velocity of light in a body moving with velocity $v$ with respect to the absolute ether is $w \pm (1 - 1/n^2)v$. Here $w$ (which equals $c/n$) would be the light velocity were the body at rest. The value $w \pm (1 - 1/n^2)v$ is the absolute velocity of propagation. The velocity of propagation with respect to the body itself is therefore $w \mp v/n^2$. Let us consider the case for light propagating in the direction opposed to the ether wind. The velocity is then $w - v/n^2$. The time $t$ required for the light to travel through a body of thickness $l$ is therefore

$$t = l/(w - v/n^2) \tag{5}$$

If $v/w$ is much smaller than unity, we can write Equation (5) to the first approximation as

$$\begin{aligned} t &\approx (l/w)(1 + v/wn^2) \\ &\approx l/w + lv/w^2n^2 \\ &\approx l/w + lv/c^2 \end{aligned} \tag{6}$$

[7] Mascart, "Les modifications (2ème partie)," p. 420.

[8] W. Veltmann, "Fresnel's Hypothese zur Erklärung der Aberrationserscheinungen," *Astronomische Nachrichten*, 1870, *75*:145–160, "Ueber die Fortpflanzung des Lichts in bewegten Medien," *Astron. Nachr.*, 1870, *76*:129–144, "Ueber die Fortpflanzung des Lichts in bewegten Medien," *Poggendorff's Annalen der Physik und Chemie*, 1873, *150*:497–535.

[9] Alfred Potier, "Conséquences de la formule de Fresnel relative à l'entrainement de l'éther par les milleux transparents," *Journal de Physique (Paris)*, 1874, *3*:201–204.

Terms of the second and higher orders of $v/w$ are neglected. Now $l/w$ is the time required for the light to travel the distance $l$ were the ether in the body at rest. The effect of the ether drag is to increase the transit time by $lv/c^2$ owing to the motion of the body.

Now let us examine Fresnel's hypothesis in conjunction with Fermat's principle. Let *ABCDEF* be the trajectory of a ray which has undergone reflections or refractions at *BCDE* in a medium at rest with respect to the ether. If *AB'C'D'E'F* represents a trajectory which is infinitesimally near *ABCDEF*, and with the same end points, Fermat's principle states that the time needed to traverse the unprimed path is less than that for all primed paths. What is the effect of imposing a velocity $v$ with respect to the ether on the medium? Let the elements of the path *AB*, *BC*, etc. be designated as $l_1, l_2, l_3 \ldots$ The effect of the motion is to increase the time for the light to traverse each element by $\mathbf{l}_i \cdot \mathbf{v}/c^2$. The total time increase for the path *AB* is therefore the sum $\Sigma \mathbf{l}_i \cdot \mathbf{v}/c^2$ which may be written as $\mathbf{L} \cdot \mathbf{v}/c^2$ where $L$ is the distance from $A$ to $F$. It is obvious that the effect of the motion on the time required for the path *AB'C'D'E'F* is an increase of exactly the same amount $\mathbf{L} \cdot \mathbf{v}/c^2$. Therefore that path which took the least time from $A$ to $B$ for a stationary medium remains the least-time path when the medium moves, since the times for all paths are increased by the same amount $\mathbf{L} \cdot \mathbf{v}/c^2$.

This result immediately accounts for the null result of the Arago experiment. Moreover it means that interference phenomena are unaffected by motion. For example, let *ABC* and *AB'C* be the two paths determined by an interferometer in going from $A$ to $C$. For simplicity consider the entire apparatus encased in the medium $M$. If the medium were at rest, a fringe pattern would result. Since motion of the medium would increase the time of travel for each path by the same amount, the phase difference between the two rays arriving at $C$ would be unchanged by the motion. Therefore no fringe shift would occur. This is Potier's interpretation of the Mascart-Jamin experiment.

Although Potier, in his paper, refers to Veltmann's proposition, I believe Potier deserves at least as much credit as Veltmann. His proof is lucid, elegant, and more general than Veltmann's. Moreover he points out explicitly that it is valid to the first order in $v/w$ only. If higher-order terms are considered, nonvanishing interference effects should occur according to Potier.

Potier's result also foreshadows the Lorentz transformation in a curious way. The increase of time $\mathbf{L} \cdot \mathbf{v}/c^2$ is reminiscent of the $xv/c^2$ term in the Lorentz time transformation. However, there is no recognition of the relativity of simultaneity nor any introduction of the $\gamma$ factor, $(1 - v^2/c^2)^{-\frac{1}{2}}$.

H. A. Lorentz knew of these ideas, for he cited Veltmann's 1873 paper in his own 1895 treatise on electrons in a motionless ether.[10] This is perhaps important for Einstein's development, as Einstein was aware of Lorentz' 1895 book, which he acknowledged in a letter to Carl Seelig, as quoted by Born.[11]

The immediate effect of the work of Mascart, Veltmann, and Potier was the abandonment of the search for first-order effects. In spite of the difficulty with dispersion, Fizeau's experiment and all the experiments culminating with Mascart had established Fresnel's drag hypothesis. The belief in the hypothesis plus the Potier-Veltmann principle stimulated a search for experiments which would demonstrate second-order

[10] Hendrik Antoon Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern* (Leiden: Brill, 1895). This book has been republished in Lorentz's *Collected Papers* (The Hague: Nijhoff, 1937), Vol. V, p. 1. The discussion of Veltmann's ideas appears on p. 102.

[11] Max Born, "Physics and Relativity," *Helvetica Physica Acta*, Supplementum IV, 1956: 244–260. Seelig's letter is on p. 248.

$(v/c)^2$ effects. The most famous of these is the Michelson-Morley experiment. It is amusing (and perhaps significant) to note that Michelson, when reporting his first experiments to the Académie des Sciences of France in 1882, made an error in his analysis which was corrected by Potier, who was in the audience.[12]

### 6. Fresnel drag in terms of the special theory of relativity

The special theory of relativity denies the existence of both an absolute frame of reference and the ether itself. It also denies an absolute time and asserts that each inertial or Galilean frame has its own time. Since there is no ether, there can be no ether drag. The results of Type II experiments are essentially a tautology. All components of such an experiment are at rest in the laboratory. Denial of the ether wind implies the absence of any motion apart from that of the light itself. An experiment such as that of Michelson and Morley must give a null result, for the isotropy of the propagation makes the two arms of the interferometer equivalent.

The Type I experiments are somewhat different. The Fizeau experiment did indeed confirm the Fresnel drag. Thus if the special theory is to be correct, it must account for the Fizeau experiment. Max von Laue showed that the special theory and the Fizeau experiment were not inconsistent.[13] The result followed from the Einstein velocity addition law. Let the water move with velocity $v$ with respect to the laboratory. The velocity of light with respect to the water is $c/n$. If we do not assume an ether, a Galilean velocity addition law would give the light velocity $c_l$ in the laboratory as $c/n + v$. For propagation parallel to the direction of relative velocity the Einstein velocity addition law gives, however,

$$\begin{aligned} c_l &= (c/n + v)/(1 + vc/nc^2) \\ &= (c/n)(1 + nv/c)(1 + v/nc)^{-1} \end{aligned} \tag{7}$$

If we expand this expression and neglect terms of the order of $(v/c)^2$ and higher, we obtain

$$\begin{aligned} c_l &\approx (c/n)(1 + nv/c)(1 - v/nc) \\ &\approx c/n + (1 - 1/n^2)v + \dots \end{aligned} \tag{8}$$

This is the result measured by Fizeau.

### 7. The Fresnel drag formula as the generator of the Lorentz group

Despite Fresnel's assumption of an absolute ether, his law may be considered a statement of the principle of relativity. This was shown definitively quite recently by Jean Abelé and Pierre Malvaux,[14] whose argument has been reproduced by Olivier Costa de Beauregard.[15] Setting the velocity of light *in vacuo* equal to unity, they write the Fresnel law as

$$w = u + v(1 - u^2) \tag{9}$$

Here $u$ is the velocity of light with respect to the medium, $v$ is the velocity of the medium with respect to the laboratory, and $w$ is the velocity of light with respect to the laboratory. They have postulated the Fresnel drag law to be the infinitesimal form of a

[12] Loyd S. Swenson, Jr., "The Michelson-Morley-Miller Experiments before and after 1905," *Journal for the History of Astronomy*, 1970, *1*:56–78.

[13] Max von Laue, "Die Mitführung des Lichts durch bewegte Körper nach dem Relativitätsprinzip," *Ann. Phys.*, 1907, *23*:989–990.

[14] Jean Abelé and Pierre Malvaux, *Vitesse et univers relativiste* (Paris: SEDES, 1954), n. 2, par. 4, pp. 156–158.

[15] Olivier Costa de Beauregard, *Précis of Special Relativity* (New York/London: Academic Press, 1966), Sec. 2.8, p. 33.

velocity addition law, since $v$ is small compared with $w$ and $u$. If this is the generating formula of a continuous connected group with an ordered set of velocities, the composition law of velocities expresses closure under addition,

$$F(w) = F(u) + F(v) \tag{10}$$

Since $w$ equals $u$, for $v$ equal to zero,

$$F(0) = 0 \tag{11}$$

Therefore $F(0)$ represents the identity element of the group. Moreover, from our initial assumption that Equation (9) is an infinitesimal form of the law, we can consider it to be the first term in a Taylor series expansion of $w$ in powers of $v$. This assumption enables us to determine the function $F$ by recognizing $(1 - u^2)$ as the coefficient of the first power of $v$ in the series expansion. We therefore write

$$1 - u^2 = \partial/\partial v\,[w(u, 0)] = [F'(v)/F'(w)]_{v=o} = F'(0)/F'(u) \tag{12}$$

since $F'(w)\mid_{v=0}$ equals $F'(u)$ from Equation (10). Therefore

$$F'(u) = A/(1 - u^2) \tag{13}$$

$$F(u) = (A/2)\log[(1 + u)/(1 - u)]$$

where $A$ is constant. Inserting this result in Equation (10) we find

$$(1 + w)/(1 - w) = [(1 + u)/(1 - \mathrm{u})]\,[(1 + v)/(1 - v)] \tag{14}$$

or

$$w = (u + v)/(1 + uv)$$

which is precisely the Einstein velocity addition law for $u$ parallel to $v$.

Abelé and Malvaux have summarized the significance of this derivation: "Once the group structure is postulated, Fizeau's experiment teaches us that there is an upper limit to all velocities, common to those of matter as well as light."[16]

It is therefore not surprising that the Potier-Veltmann principle seems to foreshadow the Lorentz transformations. Once the Fresnel law enters, the principle of relativity is present implicitly. For Fresnel unwittingly built Einstein's principle of relativity into the drag law.

[16] Abelé and Malvaux, *Vitesse*, p. 156.