

An Experimental Study of the Rate of a Moving Atomic Clock. II

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INTRODUCTION

IN the first paper under this title¹ it was shown, by experiments on high speed hydrogen canal rays, that the frequency of a moving radiating source is altered by the factor $(1 - v^2/c^2)^{1/2}$, where v is the velocity of the source, and c the velocity of light. It is the purpose of this paper to describe experiments performed with the same apparatus, having for their object, first, to increase greatly the range of velocities, and second, to investigate the influence of certain factors which, it has been suggested, might have significantly affected the results previously obtained.

EXTENSION OF VELOCITY RANGE

It was noted in the presentation of the previous results that the largest shift of the center of gravity of the Doppler lines was only about half the separation of the two components of the line ($H\beta$) used, so that the result might conceivably be due to a shift of intensities between these components, under the conditions holding in the processes of light production. A shift of at least twice that previously obtained was therefore set as the goal, calling for double the voltage between the canal ray tube electrodes, or, as an alternative, the observation of the single mass, H_1 , particles. Production of light from H_1 particles has never been observed in our experiments, in spite of a wide variation of pressure and other conditions, so that resort had to be made to increase of speed of the H_2 and H_3 particles.

The chief difficulty encountered when the voltage is raised above about 20,000 is breakdown and sparking between the accelerating electrodes, which not only prevents the establishment of higher voltages, but, if occurring often, punctures and ruins the glass wall of the tube. This difficulty was finally overcome by the use of multiple electrodes, between any two of which the voltage

was kept below the critical figure. In Fig. 1 is shown an electrode structure of the type used in the later experiments with these gaps. The first electrode nearest the arc is of iron, the others of aluminum. Besides dividing the total voltage drop this arrangement has the advantage that, by proper choice of the first electrode voltage, the canal ray stream can be focused so as to be of very uniform cross section. For this purpose the first gap should have a voltage drop of about 5000, and total voltages, for a three-gap tube, up to about 43,000 were obtained by distributing the difference over the second and third gaps.

During the period of development of these tubes the voltage used was gradually pushed up to the desired figure, and a series of observations accumulated, which are listed in Table I and shown in Fig. 2. These were made in the same way as those described in the first paper, except that they were all made on the rapid, coarser grained plates, which, by the shorter exposures, reduced the otherwise high mortality of filaments and tubes. This procedure resulted in somewhat less precision in the measurements, which is offset by the comparatively large number and range of points obtained.

Inspection of Fig. 2 shows that the desired goal of doubling the observed shift was attained, and that over this entire range the relationship previously observed is maintained. The highest points are at voltages around 43,000, which for the H_2 particles give a shift of over 0.11A as against the previous maximum of 0.047A. All except the lowest of the new points are for H_2 particles. This is because at these higher voltages the H_3 emission is very faint, unless the gas pressure is considerably raised, and this brings up the second spectrum in such strength that it is very difficult to find voltages at which the H_3 lines are definitely in clear spaces.

The behavior of heavier particles is, however, covered by the fortunate occurrence under certain conditions (traces of water vapor in the tube) of the line which is shown by its position

¹H. E. Ives and G. R. Stilwell, *J. Opt. Soc. Am.* **28**, 215 (1938).

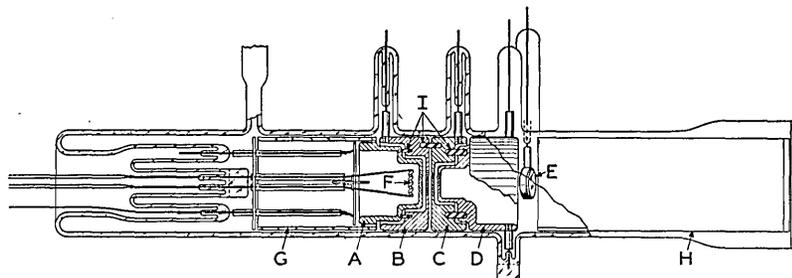


FIG. 1. Multiple gap electrode tube used to obtain high accelerating voltages. *A*—iron accelerating electrode; *B*, *C*, *D*—aluminum perforated electrodes; *E*—concave mirror; *F*—filament; *G*, *H*—shielding glass inner tubes; *I*—quartz electrode separators.

to be due to H_3O . This compound, of mass 19, gives an emission line at about one-third the distance from the center line of the H_2 particles, too close for accurate measurements at any but the highest voltages. Figure 3 is a reproduction of a negative made at 42,280 volts showing the suppression of the H_3 lines, above noted, and the occurrence of the H_3O lines. A measurement of the H_3O line from this negative is shown on Fig. 2, where it is almost coincident with the lowest H_3 point previously obtained, in a region very free of second spectrum lines, and falls accurately on the theoretical curve. This H_3O point and the lowest of the new series on H_2 and H_3 practically span the entire region covered by the earlier measurements.

Also plotted in this figure along their appropriate theoretical curve are three values, obtained for H_3 particles by Otting² who used the red hydrogen line, 6563A and measured the wave-lengths by means of an interferometer arrangement in front of the spectroscope slit. These effectively supplement our own low voltage values, given in the previous paper.

DOPPLER EFFECT IN "UNDISPLACED" CENTRAL REFERENCE LINE

In a critical study of the theory of this experiment, R. C. Jones³ has suggested that the energy imparted to the "stationary" particles in the canal ray tube may produce a velocity of these in the direction of motion of the moving particles. Such a velocity would shift the central reference line toward the blue, thus introducing an apparent shift of the displaced lines toward the red. Jones indicates how, on certain assumptions, this shift might be of the order of magnitude of the observed effects.

² G. Otting, *Physik. Zeits.* 40, 681-7 (1939).

³ R. C. Jones, *J. Opt. Soc. Am.* 29, 337 (1939).

Against the possibility of such an effect being significant in the experiment we have, first, the consideration that the central line is not only derived from the directly observed light, but from that reflected in the mirror, so that any such shift would result in a doubling of the line; or if the shift is small, in a mere broadening, which will be substantially symmetrical with an aluminum coated mirror of 80-90 percent reflecting power, such as that used. Second, if the effect were due to a shift of the central line it would result in *equal* shifts for the H_2 and H_3 lines produced by the same voltage on the tube, which is not the case.

The point raised by Jones is, however, of sufficient theoretical interest so that experimental arrangements were devised to permit its investigation. For this purpose a special fixed aperture slit was constructed, with jaws made of two inclined razor blades, and furnished with two front surface mirrors at 45° , so that a three-part field was produced, the upper and lower portions of which could be illuminated through reflecting prisms by an external comparison source. A drawing of this slit is shown in Fig. 4. For the external source a low voltage hydrogen

TABLE I.

PLATE	APPROXIMATE VOLTAGE	PARTICLE OBSERVED	OBSERVED MEAN $\Delta\lambda$	$\lambda_0(\frac{1}{2}v^2/c^2)$ COMPUTED FROM $\Delta\lambda$	$\Delta\lambda$ OBSERVED
214	26735	H_2	25.82	0.0670	0.067
		H_3	21.08	.0457	.045
215	27270	H_2	26.05	.0686	.0675
216	28185	H_2	26.53	.0724	.080
218	28185	H_2	26.56	.0725	.0755
222	34395	H_2	29.40	.0869	.090
223	40190	H_2	31.93	.1049	.0995
228	41640	H_2	32.01	.1054	.1145
230	42260	H_2	32.50	.1098	.113
231	42280	H_2	32.29	.1073	.1145
		H_3O	10.72	.0012	.012

arc was employed, illuminating a diffusing glass close to the 45° prisms.

The canal ray tubes used in this study were made double ended, with two sets of electrodes, as shown in Fig. 5, so that the discharge could be observed both approaching and receding. The concave mirror was furnished with a cap which could be made, by external magnetic control, to cover or expose the mirror at will and was viewed by reflection in a second plane mirror so placed as to permit observation from the side of the tube without interference by the electrodes.

Preliminary measurements of spectrograms of the central or undisplaced line showed that any effect, if present, was too small to be definitely established by the plate measuring procedure previously employed. Accordingly, a special procedure was resorted to, involving densitometric measurements of the greatly magnified image. The apparatus for this is shown by the drawing, Fig. 6. The three-part image of the central line

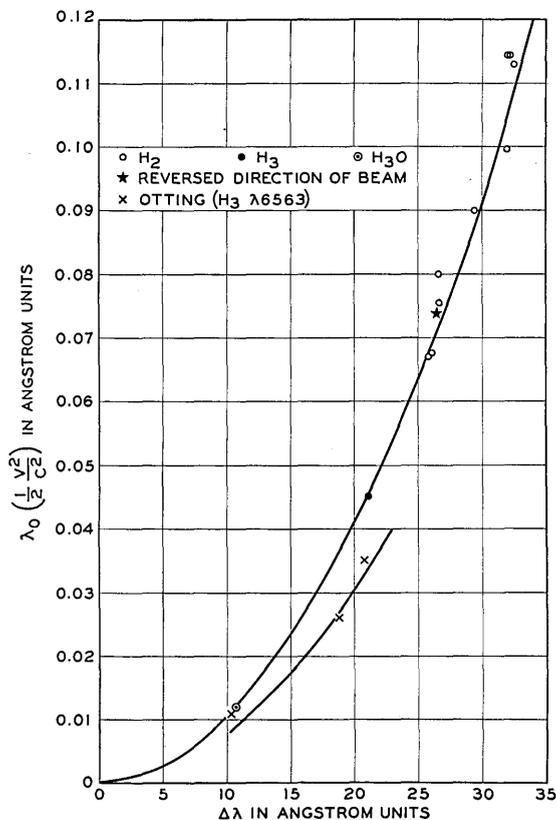


FIG. 2. Plot of new observations or shift of center of gravity of approaching and receding Doppler lines.

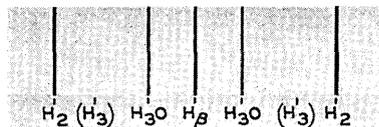


FIG. 3. High voltage negative showing suppression of H_3 lines, and occurrence of H_3O line due to water vapor.

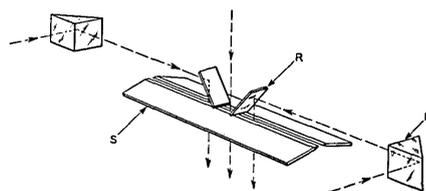


FIG. 4. Three-part fixed slit for studying behavior of "undisplaced" line. *S*—slit with razor blade jaws; *R*—first surface mirrors for introducing light from comparison source; *P*—prisms.

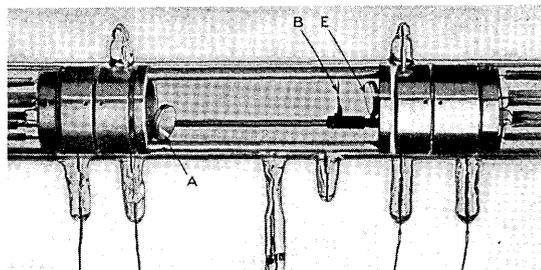


FIG. 5. Double-ended canal ray tube. *E*—concave mirror; *B*—movable obstructing plate; *A*—mirror for observing canal ray beam from side of tube.

and comparison source was illuminated by the image of the glowing electrode of a Pointolite lamp focused upon it. The resultant image was then projected by means of a high quality (Tessar) photographic lens of 25-mm focal length onto a large three-part slit of 0.25-mm opening, backed up by diffusing plates, behind which were three photoelectric cells in separate shielded compartments. A triple shutter permitted exposure of any cell at will, and the slit and cells were mounted on a platform movable by a micrometer screw. The magnification was such that one millimeter motion of the slit corresponded to 0.1A on the plate. The intensity of the Pointolite source was indicated by a separate photoelectric cell and galvanometer with an extended scale, and could be accurately held constant by a series resistance.

The procedure in making the three-part negatives was so to adjust the illumination from the

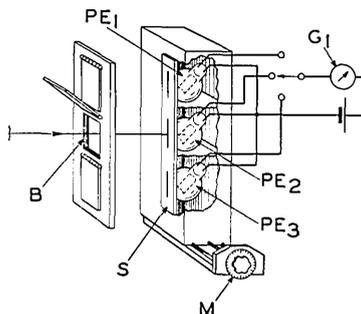
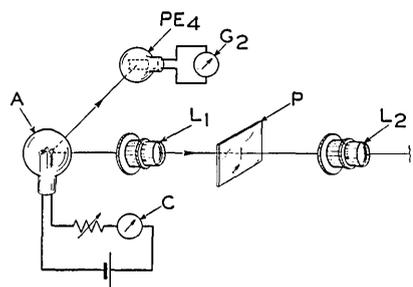


FIG. 6. Apparatus for measuring density distribution in canal ray spectrograms. *A*—Pointolite lamp; *B*—triple shutter; *C*—current control for Pointolite lamp; *P*—spectrogram to be measured; *L*₁—lens for imaging Pointolite on plate; *L*₂—lens for imaging spectrogram on photo-cell slit; *S*—three-part photo-cell slit; *PE*₁, *PE*₂, *PE*₃—photoelectric cells for measuring image on *S*; *G*₁—galvanometer for measuring photo-cell currents; *PE*₄, *G*₂—photo-cell and galvanometer for monitoring Pointolite intensity; *M*—micrometer screw for moving photo-cells across spectrogram image.

comparison source that it gave images closely of the same density as that from the canal rays. A positive print made from the negative was mounted in the projection apparatus with the three images falling closely along the line of the slit over the photoelectric cells and was measured by the three cells in succession at intervals of 0.25 mm along the magnified image. The appearance of the projected positive image and its placement over the photo-cell slits is shown in Fig. 7. In Fig. 8 is shown a typical set of readings of photo-cell currents against distances along the spectrograph image with their graphically determined centers of gravity. The positions of the comparison lines so determined were joined by a straight line, as shown in Fig. 9 and the position of the central image, plotted at the mid-ordinate, showed at once its displacement. A complete measuring procedure, in order to eliminate all irregularities of the slits and distortion of the lens, would call for reversing the spectrograph slit, and inverting the plate in the measuring apparatus. Such multiple measurements were, however, found to be unnecessary, as measurements so made on negatives in which the three images all came from the comparison source showed the slits to be accurately straight and no lens distortion to be present within the errors of measurement.

The results of these tests may be summarized as follows: With the mirror in operation the central image shows no displacement within the error of measurement.⁴ With the mirror covered,

a very small displacement appears, showing that the "undisplaced" particles have a slight velocity imparted to them in the direction of motion of the moving particles. A tabulation of a series of four measurements made at 14,000 volts is given in Table II.

The velocities imparted to the "stationary particles" at this voltage are from these figures about 750 meters per second. It follows from these results that if the mirror is for any reason of low reflecting power, as from tarnishing, the observed red shifts may be expected to be too large. The slight tendency of the plotted points in Fig. 2 to be above the theoretical curve may

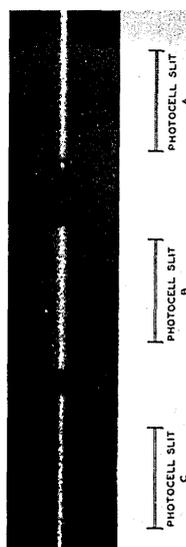


FIG. 7. Positive from spectrogram as projected on triple slit over photo-cells.

⁴ In the experiment as performed by Otting (reference 2), in which the wave-lengths were measured with reference to

two neon lines, the central hydrogen canal ray line, as formed by combined direct and reflected light, was found to be unshifted, as compared with the line from a Geissler tube.

be due to the mirror being less than 100 percent reflecting power.

INTERCHANGE OF DIRECT AND REFLECTED COMPONENTS

In the canal ray tubes as used in the experiments previously reported, the particles approaching the spectrograph were observed directly, those receding from it by reflection in a concave mirror placed at a distance from the spectrograph slit equal to the radius of curvature of the mirror. Variations from this mirror arrangement were made which were held to establish that no significant dissymmetry resulted from its use (see reference 3 of the first paper). In the paper by Otting (reference 2), this optical arrangement is criticized, with the implication that it invalidates the result. While

we cannot grant the soundness of this criticism on optical grounds, it has been thought worthwhile to show its invalidity by the experiment of reversing the direction of the canal rays with respect to the mirror, so that the receding rays are directly observed, and vice versa.

This experiment is simply done by use of the double-ended canal ray tube described in the last section (Fig. 5), the direct observation being made effectively along the line of travel of *receding* particles, the concave mirror now reflecting the light as from *approaching* particles, this being the opposite of the conditions in the other experiments cited. It is now the Doppler line on the blue side which comes from the concave mirror, while before it was the Doppler line on the red side.

In this test double-gap electrodes were used, permitting a voltage of approximately 28,300, with the result summarized in Table III, for

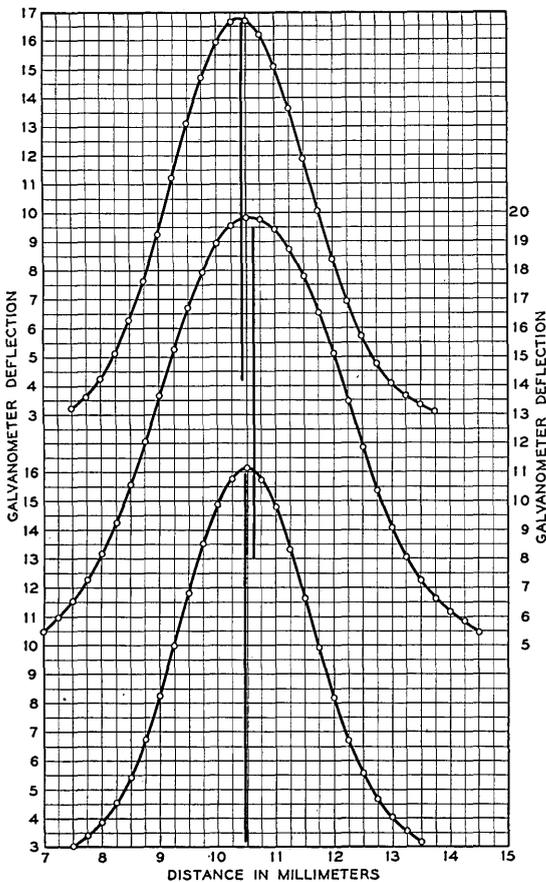


FIG. 8. Typical measurements of transmission of spectrogram made with three-part slit.

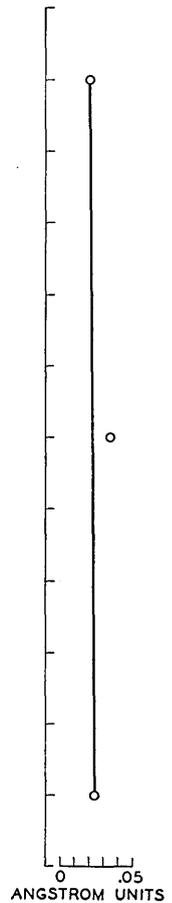


FIG. 9. Determination of shift of center line from densitometer measurements.

TABLE II.

	PARTICLES APPROACHING SLIT. SHIFT TOWARD BLUE	PARTICLES RECEDING FROM SLIT. SHIFT TOWARD RED
No Mirror	0.0125A	0.0125A
Mirror In	0.002A	0.0000A

TABLE III.

VOLTAGE	$\Delta\lambda$ OBSERVED	$\frac{1}{2}(v^2/c^2) \cdot \lambda$ COMPUTED	$\frac{1}{2}(v^2/c^2) \cdot \lambda$ OBSERVED
28,300	26.40	0.0717A	0.074A

H₂ particles. The point so fixed, shown by the star on the plot of all observations (Fig. 2), shows conclusively that the observed frequency changes are not significantly affected by the optical arrangement used for securing simultaneous observation of approaching and receding particles, and completely disposes of this criticism.

DISCUSSION

The net result of this whole series of experiments is to establish conclusively that the frequency of light emitted by moving canal rays is altered by the factor $(1 - v^2/c^2)^{1/2}$. This agrees with the prediction of Larmor and Lorentz, and, unless such a frequency change should be shown to be the result of other factors than the speed of the particles, constitutes a verification of their prediction.

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Interreflections in Rooms

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1. INTRODUCTION

THE "lumen method" of illumination calculation, developed by Harrison and Anderson^{1,2} in 1916, has been of great service to the lighting art and is used extensively today in lighting design. Further contributions to the subject have been made by Dourgnon,³ Hisano,⁴ Wetzel,⁵ Margoulis,⁶ Vallat,⁷ and others; but

¹ W. Harrison and E. A. Anderson, "Illumination efficiencies as determined in an experimental room," *Trans. I.E.S.* 11, 67 (1916).

² W. Harrison and E. A. Anderson, "Coefficients of utilization," *Trans. I.E.S.* 15, 97 (1920).

³ J. Dourgnon, "Nouvelle méthode de prédétermination des coefficients d'utilisation dans les projets d'éclairage d'espaces clos," *Gen. Elec. Rev.* 23, 271 (1928). J. Dourgnon, "Définition et calcul des grandeurs caractéristiques de l'éclairage d'un espace clos," *Gen. Elec. Rev.* 33, 579 (1933).

⁴ K. Hisano, "Light flux distribution in a rectangular parallelepiped and its simplifying scale," *Res. Elec. Lab., Tokyo*, No. 394 (1936). [In Japanese.]

⁵ J. Wetzel, "Nouvelle méthode de calcul des coefficients d'utilisation," *C.I.E. Proc.* p. 517 (1931).

⁶ W. Margoulis, "Etude nomographique pour la détermination des coefficients d'utilisation dans les projets d'éclairage," *Gen. Elec. Rev.* 24, 439 (1928).

⁷ A. Vallat, "Les méthodes d'évaluation des facteurs d'utilisation," *Comptes rendus Congrès Int. des Appl. de l'Eclairage* (Paris, 1937), p. 137.

the method as customarily used is essentially that developed by Harrison. The basis is purely empirical; the result is an average illumination on a horizontal "working plane."

Valuable as this result is, it is not sufficient for a thorough analysis of a lighting installation. Knowledge is required of the brightness distribution over the entire field of view;⁸ and quantitative information is needed on shadows and glare. These modern needs have not been satisfied by any previous method. The theoretical methods have ignored interreflections in the room, while the empirical methods (such as Harrison's) are limited by the paucity of experimental results and the difficulty of graphical presentation of functions of five or six independent variables.

Thus, in the further development of lighting design for interiors, there is need for a more comprehensive method of calculation than has been previously available. In the present paper an attempt is made to obtain the desired

⁸ H. L. Logan, "Specification points of brightness," *Trans. I.E.S.* 34, 881 (1939).