UPMC
SORBONNE UNIVERSITÉS

Master NPAC

Cosmology – Lesson 2

Academic Year 2016–2017

Problems

Q 1 — Suppose you are living inside the surface of a sphere of radius R. If you draw a circle of radius L, what is the circle circumference?

The Earth may be idealized as a perfect sphere of radius $R = 6371$ km. If you could measure distances with a precision of ± 1 m, how large should you draw a circle on Earth's surface to convince yourself that the Earth is spherical rather than flat?

Q 2 — Suppose you are still a two-dimensional being, trapped inside the surface of a sphere (of radius R). An object of width ℓ sits at a distance d (measured on the surface) from you. What angular width $\delta\theta$ will you measure? What happens when $d \longrightarrow \pi R$?

Q 3 — In cartesian coordinates, write and solve the geodesic equations for a two-dimensional flat plane and show that the solutions are straight lines.

Q 4 — For the two-dimensional sphere (previous question), do the following variable change: $(\theta, \varphi) \to (\rho, \Theta)$ with $\rho = R \sin\theta$ and $\Theta = \varphi$. Compute the metric with this new coordinate system. What is ρ?

Q 5 — In the usual spherical coordinates, the metric of a two dimensional sphere is:

$$\mathrm{d}l^2 = R^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)$$

where R is a constant. Compute the metric $g_{\mu\nu}$, the inverse metric $g^{\mu\nu}$, the Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$ and show that a great circle is a solution of the geodesic equation (you have the freedom to choose your coordinate system).

Q 6 — Do the complete calculation of all the non-zero Christoffel symbols for the FLRW metric. Deduce the Ricci tensor $R_{\mu\nu}$ (only terms R_{00} and R_{ii} for $i \in \{1, 2, 3\}$ are non-zero) and the Ricci scalar R^{α}_{α}. Using the expression of the energy-stress tensor $T_{\mu\nu}$, establish the Friedmann equations (The calculation takes some time).