Master NPAC

Cosmology – Lesson 3

Academic Year 2016–2017

Problems

Q 1 — Redo the calculations for the various universe models listed in lesson 3.

Q 2 — Calculate the age of a critical universe ($\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0} = 1$) in the limit $\Omega_{\Lambda,0} \ll \Omega_{m,0}$. Verify that for $\Omega_{\Lambda,0} > 0$, the age you obtain is larger than the age of a universe with $\Omega_{\Lambda,0} = 0$.

Q 3 — Consider an open universe ($\Omega_0 \leq 1$). Show that if $\Omega_{\Lambda,0} = 0$ all of the universe will eventually come within our horizon. Show that if $\Omega_{\Lambda,0} > 0$ the horizon will approach a finite limit beyond which we will never see.

Q 4 — (Exam of 2015-02-04) Consider a closed universe which contains only non relativistic matter: $\Omega_0 = \Omega_{m,0} > 1$.

(i) Describe briefly the properties of such a universe, its curvature, and its dynamics (a drawing may help).

(ii) Write the Friedmann equation for this universe (with the convention $a(t_0) = 1$, $R(t) = R_0 a(t)$). Compute the value a_{max} of the scaling factor a at maximum expansion.

(iii) Show that H_0, Ω_0 and the current universe curvature radius R_0 are linked by:

$$R_0 = \frac{c}{H_0} \frac{1}{\sqrt{\Omega_0 - 1}}$$

(iv) Calculate the present horizon d_{hor} and χ_{hor}. What fraction of the universe is presently within the horizon? Show that at the moment of maximum expansion the horizon includes the entire universe, *i.e.* $\chi_{\text{hor}}(a_{\text{max}}) = \pi$.

(v) Verify that the evolution of the universe may be described by the following parametric equations:

$$a(\eta) = A\,(1 - \cos\eta)$$
$$t(\eta) = B\,(\eta - \sin\eta)$$

Give the expression of A and B as functions of H_0 and Ω_0. What is the value of η at maximum expansion? Describe briefly the resulting dynamics.

(vi) Show that the age of the universe at maximal expansion is:

$$t(a_{\text{max}}) = \frac{\pi}{2H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}$$

(vii) At some time $t_1 > t(a_{\text{max}})$ during the contraction phase of this universe, an astronomer named Edwin Elbbuh discovers that all nearby galaxies have blueshifts ($-1 \leq z < 0$) proportional to their distance ; he measures as well $H_1 < 0$ and $\Omega_1 > 1$. Knowing $H_1 < 0$ and Ω_1, how much time remains between t_1 and the final Big Crunch at $t = t_{\text{crunch}}$?

Hints:

$$\int_A^B \frac{\mathrm{d}x}{\sqrt{x(1-x)}} = \Big[\arcsin(2x-1)\Big]_A^B \qquad x = \frac{\Omega_0 - 1}{\Omega_0} \times a$$

Q 5 — Some cosmologists speculate that the universe may contain a quantum field called "*quintessence*", which has a positive energy density and a negative equation-of-state parameter w. Let suppose that we are in a spatially flat universe, containing only matter ($\Omega_{m,0} \leq 1$) and quintessence with $w = -1/2$ and $\Omega_{Q,0} = 1 - \Omega_{m,0}$. At what scale a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find $a(t)$ for this universe. What is $a(t)$ when $a \ll a_{mQ}$? and when $a \gg a_{mQ}$? What is the current age of this universe, as a function of H_0 and $\Omega_{m,0}$?