UPMC SORBONNE UNIVERSITÉS

Master NPAC

Cosmology – Lesson 4

Academic Year 2016–2017

Problems

Q 1 — From $T_{\rm CMB} = 2.725\,\rm K$, compute the reduced photon density today $\Omega_{\gamma,0}\,h_{100}^2$. where

$$h_{100} = \frac{H_0}{100\,\rm km{\cdot}s^{-1}{\cdot}Mpc^{-1}}.$$

You might need:

$$\int_0^\infty \frac{x^3}{e^x - 1}\,\mathrm{d}x = \frac{\pi^4}{15}$$

For this question, give a literal expression of $\Omega_{\gamma,0}\,h_{100}^2$.

Q 2 — Using $\Omega_{m,0}\,h_{100}^2 = 0.11$ and $\Omega_{\gamma,0}\,h_{100}^2 = 2.5\times10^{-5}$, evaluate the scale factor $a_{\gamma m}$ and the redshift at which matter and photons had the same energy density.

Q 3 — What is the comoving horizon size at this redshift?

Q 4 — Calculate the temperature T_ν and the number density n_ν of the relic neutrinos. Deduce $\Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0}$, and the a_{rm} scale factor when matter and radiation had the same energy density. Assume 3 types of neutrinos, and that

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \qquad \frac{n_\nu}{n_\gamma} = \frac{3}{4}\frac{g_\nu}{g_\gamma}\left(\frac{T_\nu}{T_\gamma}\right)^3 \qquad \frac{\varepsilon_\nu}{\varepsilon_\gamma} = \frac{7}{8}\frac{g_\nu}{g_\gamma}\left(\frac{T_\nu}{T_\gamma}\right)^4$$

$$g_\nu = 1 \qquad g_\gamma = 2$$