

# ***Cosmology***

Master NPAC

*Lesson 4 :*

*Universe Thermodynamics : Thermal History  
Primordial Nucleosynthesis*

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4.1

***Thermal History  
An Overview***

# *Eras : radiation, matter, « vacuum »*

- Radiation dominated

$$\varepsilon_r(t) = \varepsilon_{r,0} a(t)^{-4}$$

$$a(t) \propto t^{1/2} \quad \varepsilon(t) \propto a^{-4} \propto t^{-2}$$

- Matter dominated

$$\varepsilon_m(t) = \varepsilon_{m,0} a(t)^{-3}$$

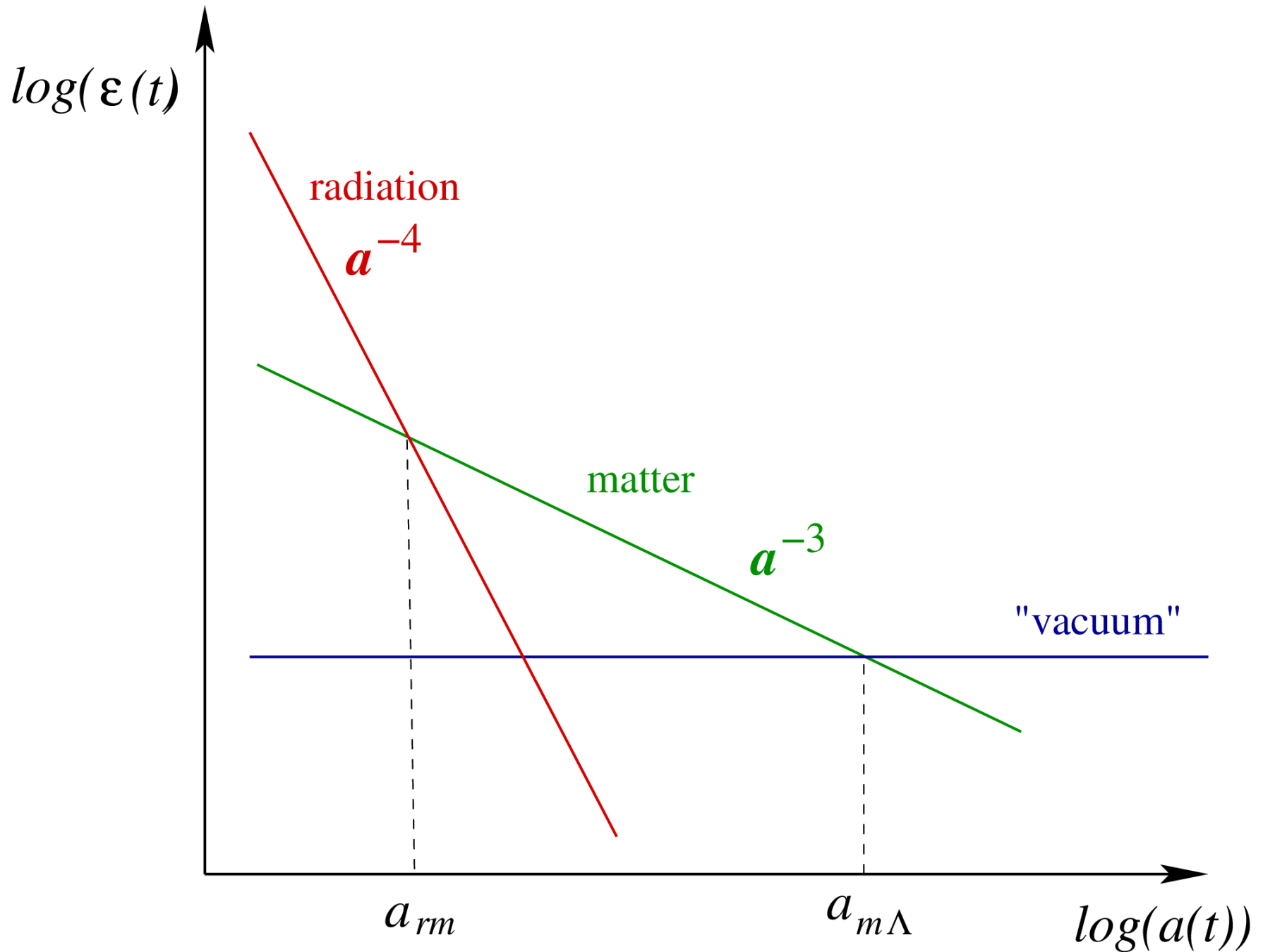
$$a(t) \propto t^{2/3} \quad \varepsilon(t) \propto a^{-3} \propto t^{-2}$$

- « vacuum » dominated  $\Lambda$

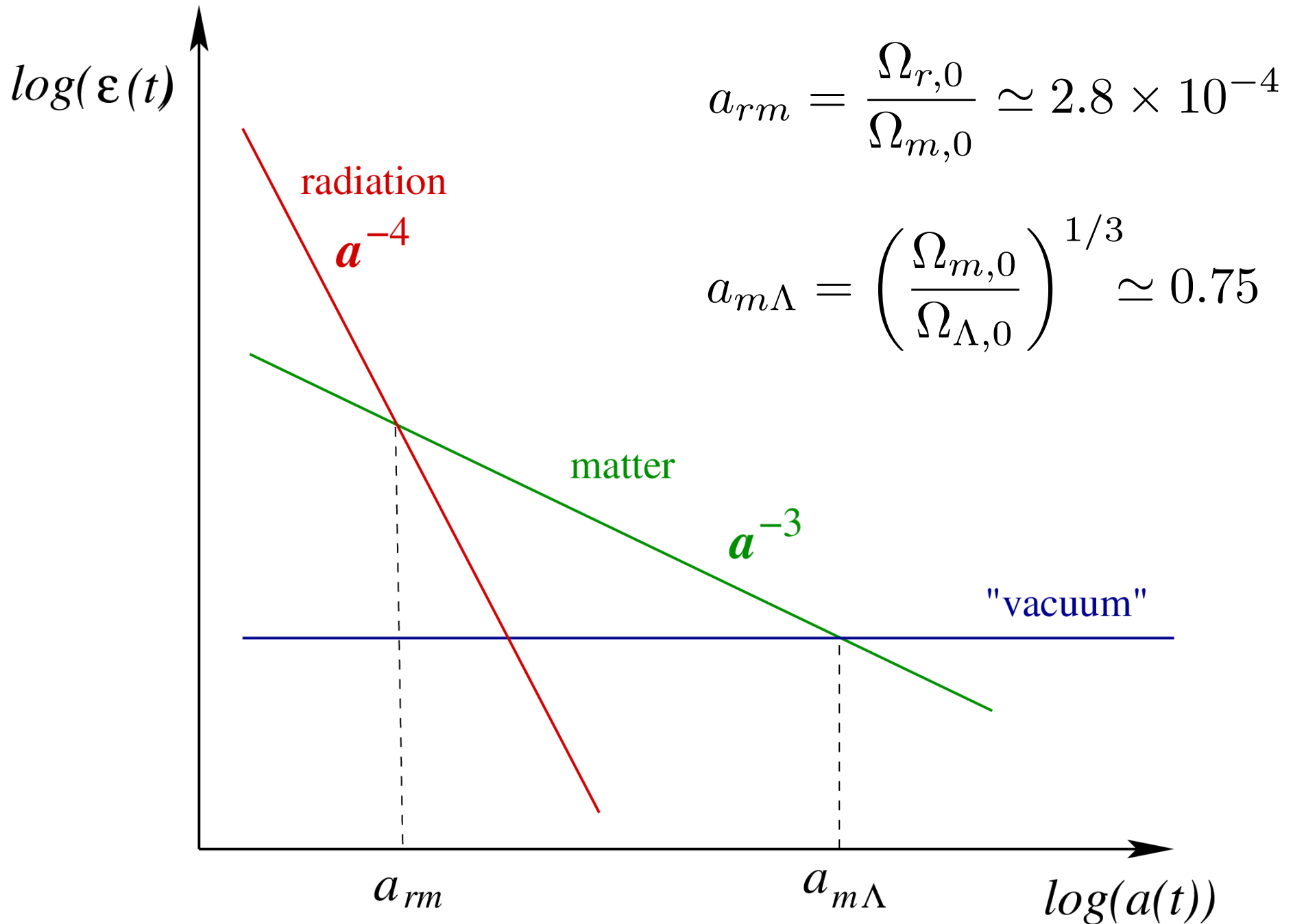
$$\varepsilon_\Lambda(t) = \varepsilon_{\Lambda,0} = \text{cste} \quad H(t) = H_0 = \text{cste}$$

$$a(t) \propto e^{H_0 t}$$

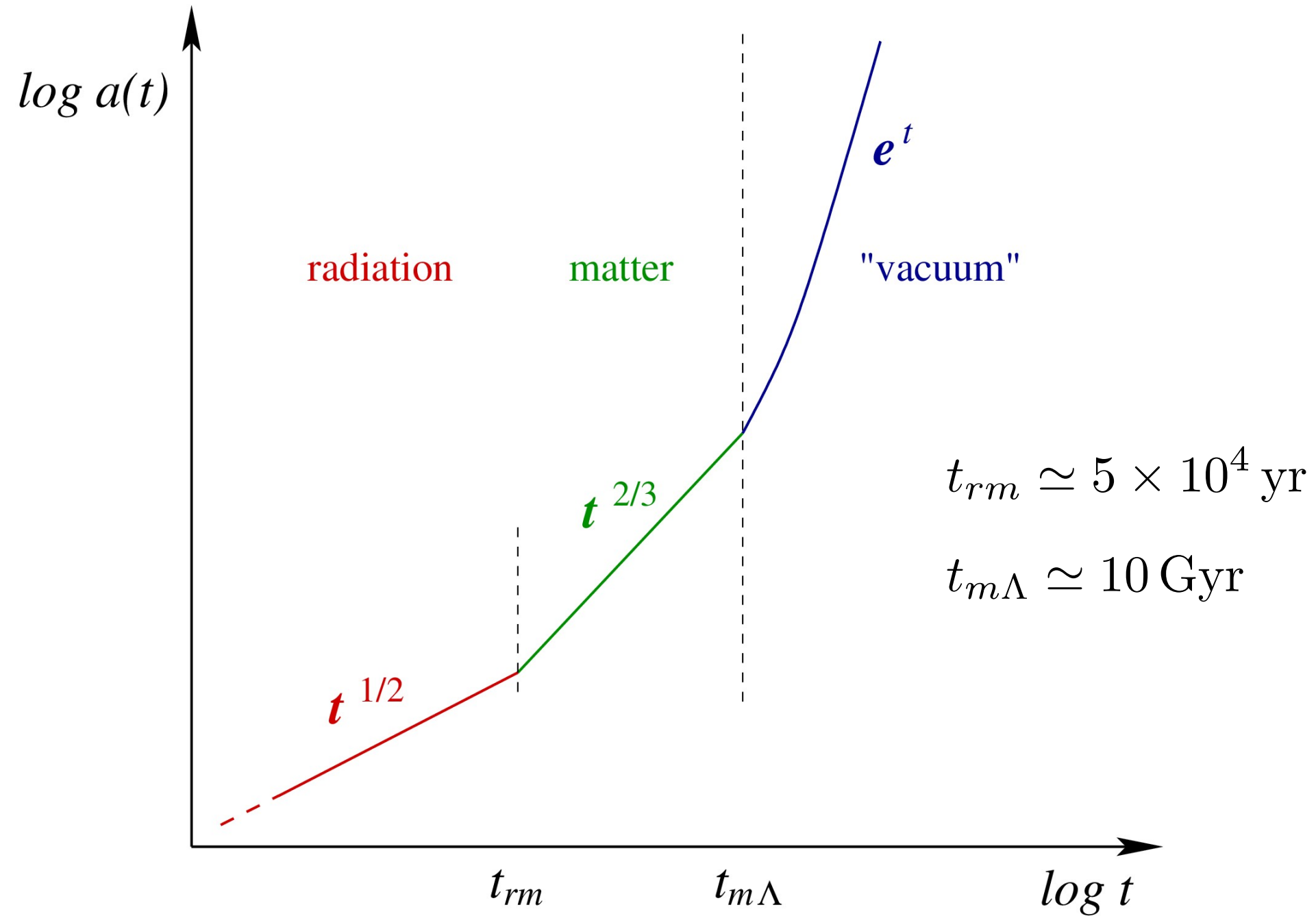
# *Eras : radiation, matter, « vacuum »*



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# Eras : radiation, matter, « vacuum »



*The Universe expands...*

$$H^2(t) = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} \varepsilon - \frac{kc^2}{R_0^2 a^2}$$

$$\frac{H^2(t)}{H_0^2} = \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\Lambda,0} + \dots + (1 - \Omega_0) a^{-2}$$

*Entropy is being conserved*  $TdS = dE + p dV = 0$

*any heat flow would defined a preferred direction → isotropy*

*The Universe expands **adiabatically**, like a fluid in equilibrium*

$$\text{Radiation } a(t) \propto t^{1/2} \quad \varepsilon(t) \propto a^{-4} \propto t^{-2} \quad T \propto a^{-1} \propto t^{-1/2}$$

*The Universe **cools down** while expanding...*

*→ **hotter and denser** in the past...*

# Cosmic Microwave Background (CMB)

A. A. Penzias & R. W. Wilson (1965)

Black Body spectrum (microwave)

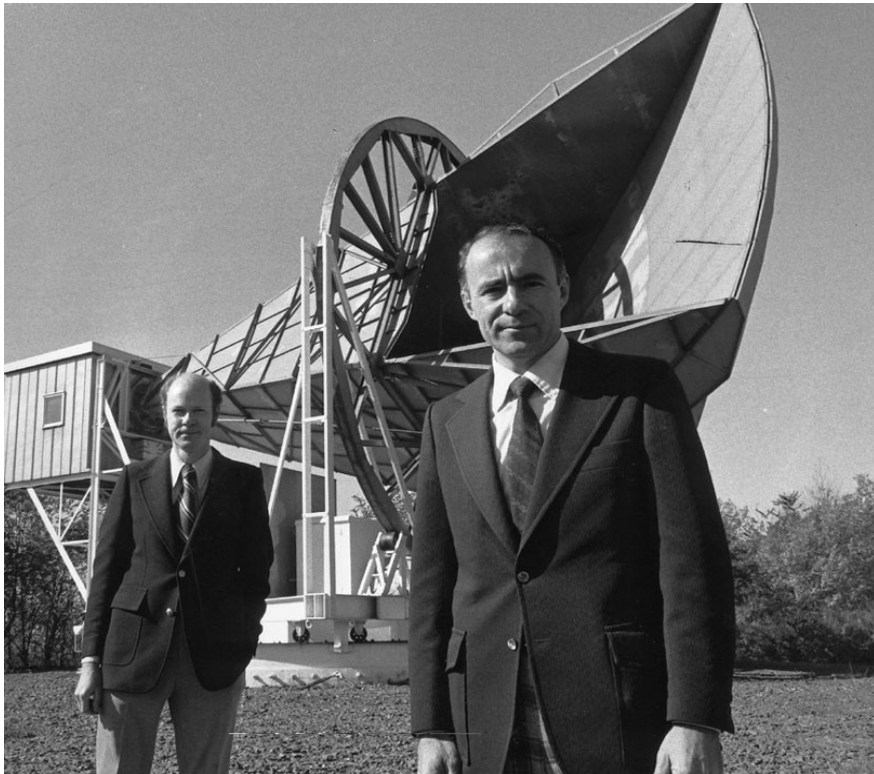
$T \sim 2.73 \text{ K}$  (BB spectrum up to  $10^{-5}$ )

Predicted by the « hot Big Bang » model

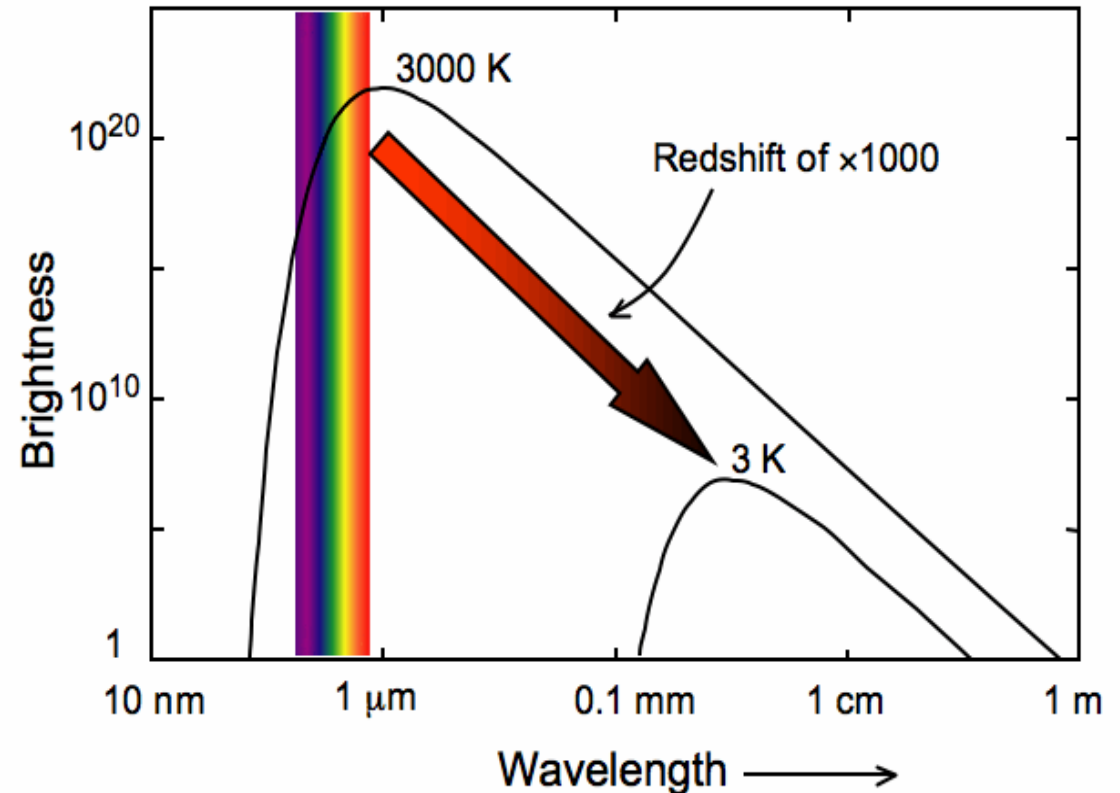
Hot Universe in the Past

Black body spectrum emitted at the time of last ionizations (decoupling of photons and matter)

$T \sim 2.73 \text{ K}$  implies a redshift of  $\sim 1100$



© 2004 Thomson - Brooks/Cole



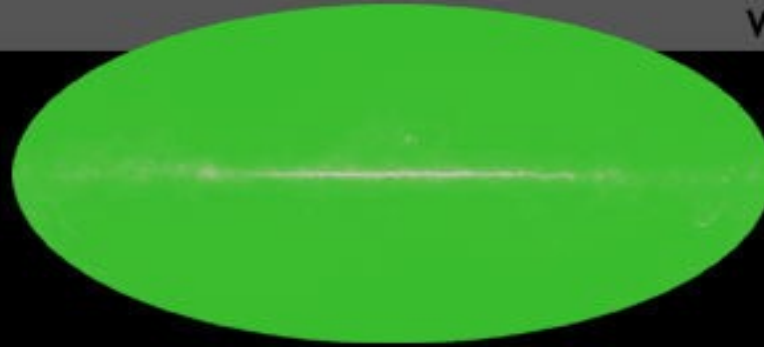


# Cosmic Microwave Background (CMB)

1965



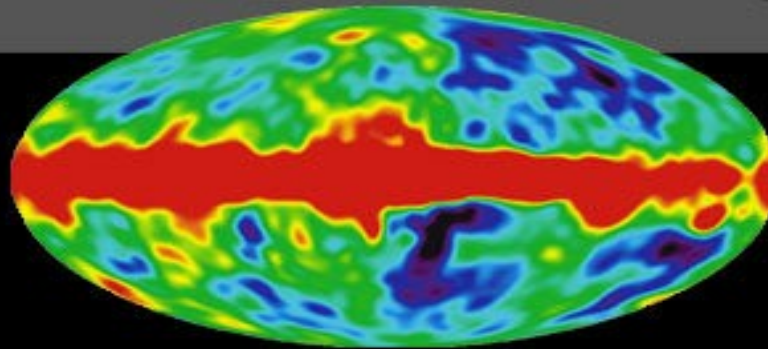
Penzias and  
Wilson



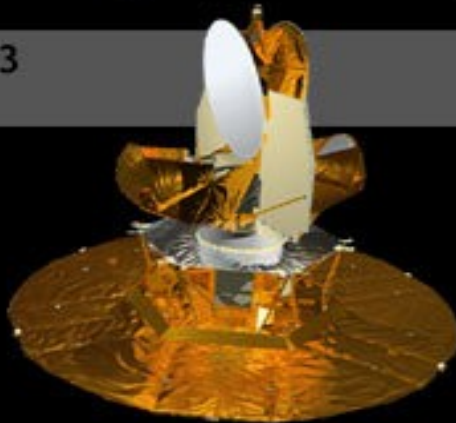
1992



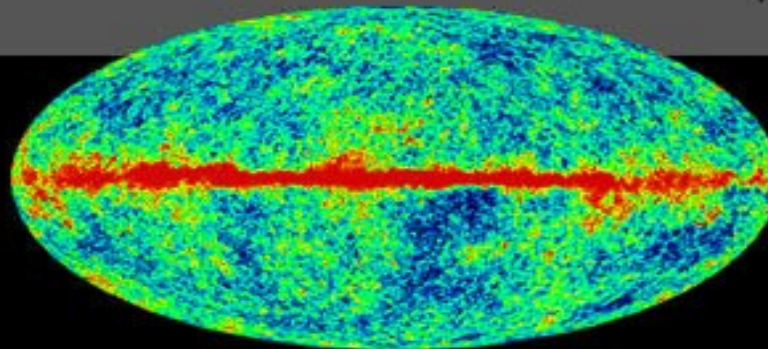
COBE



2003



WMAP

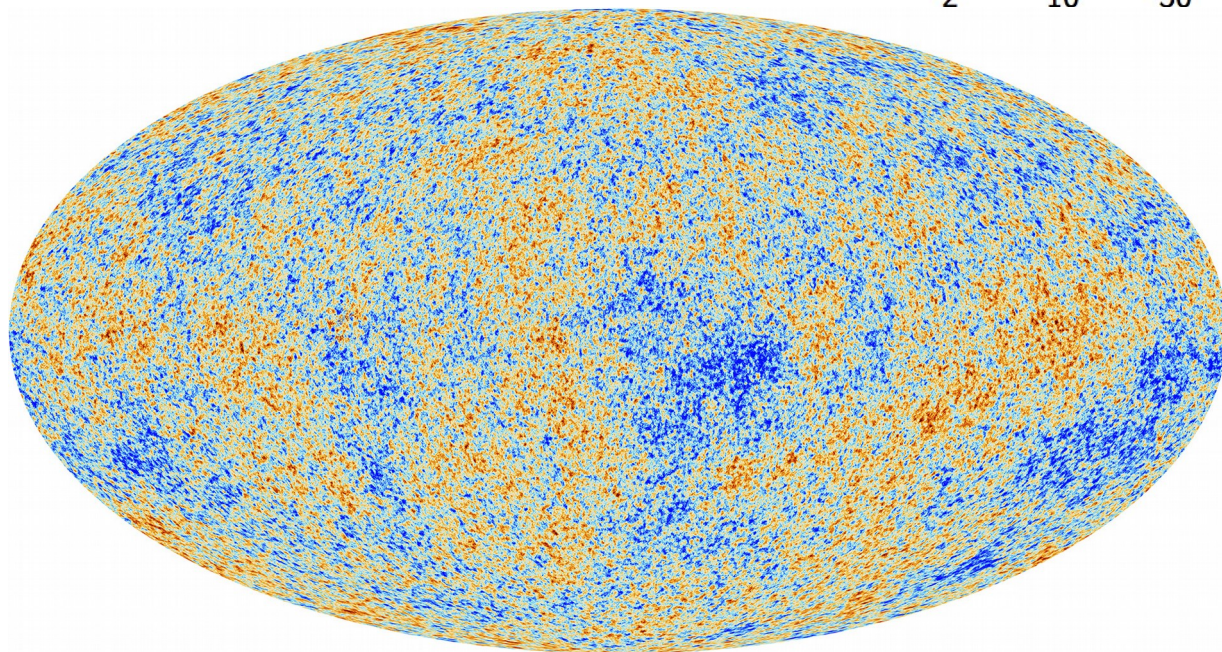
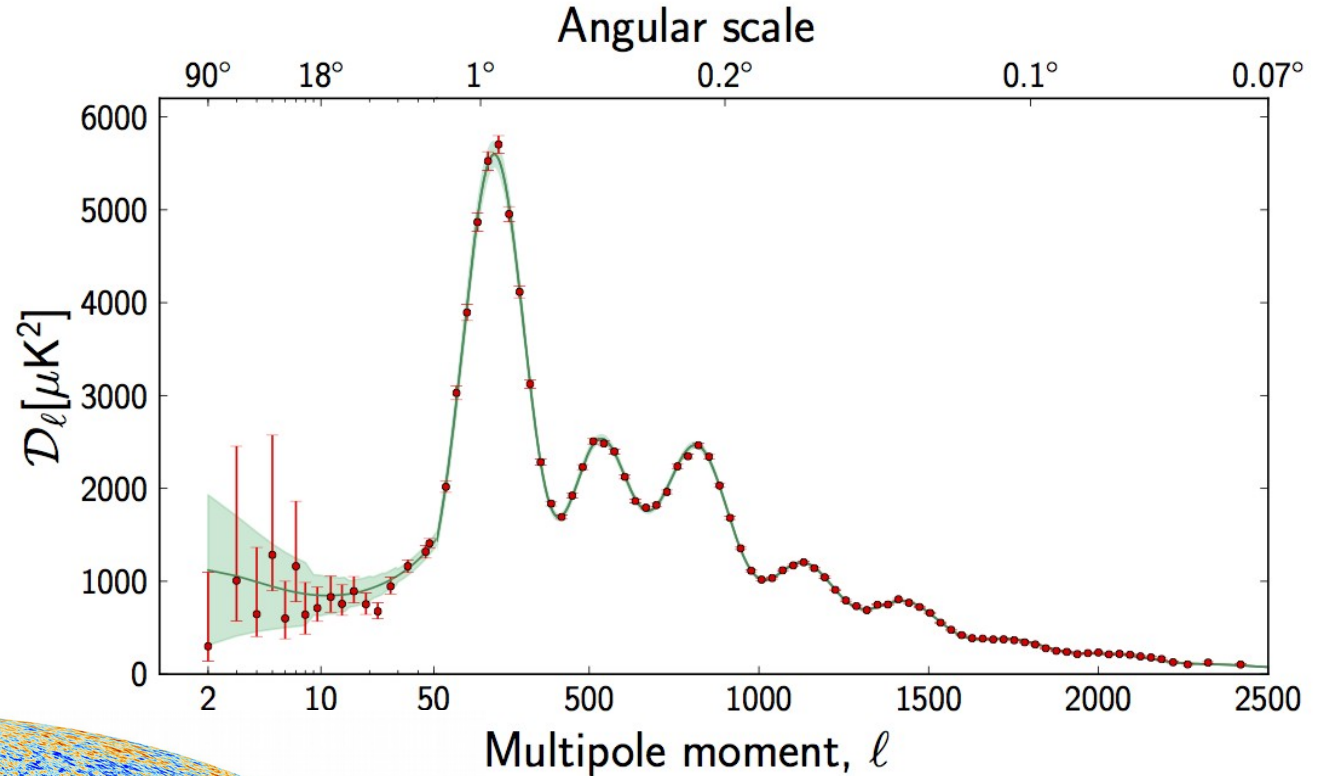


# Cosmic Microwave Background (CMB)

Anisotropies reveal  
densities fluctuations  
at the time of  
emission of the CMB

$z \sim 1100$

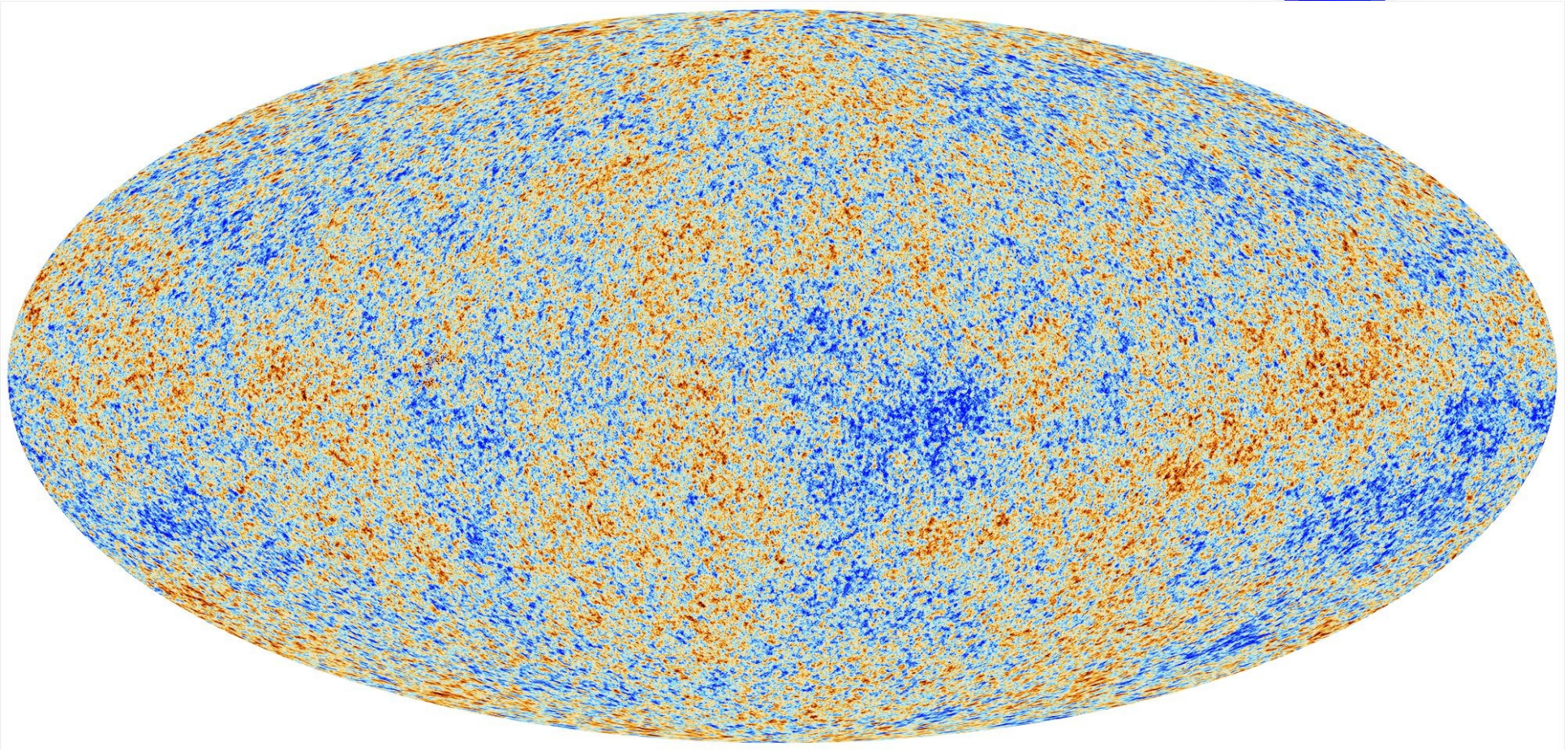
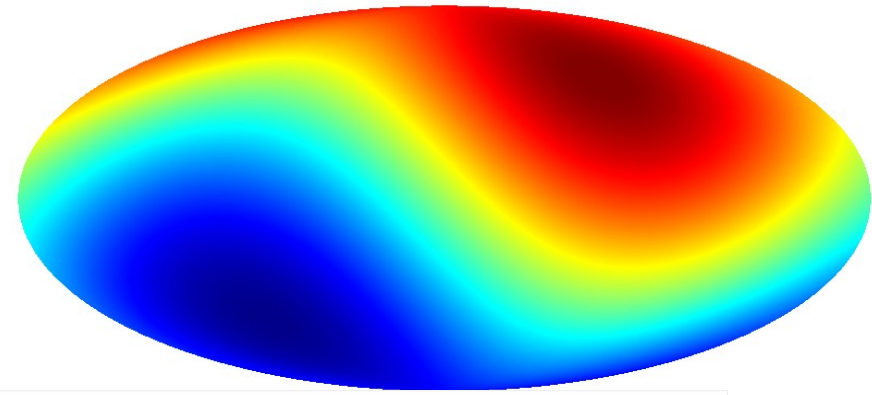
Age  $\sim 370\,000$  y



Planck  
2009-2012



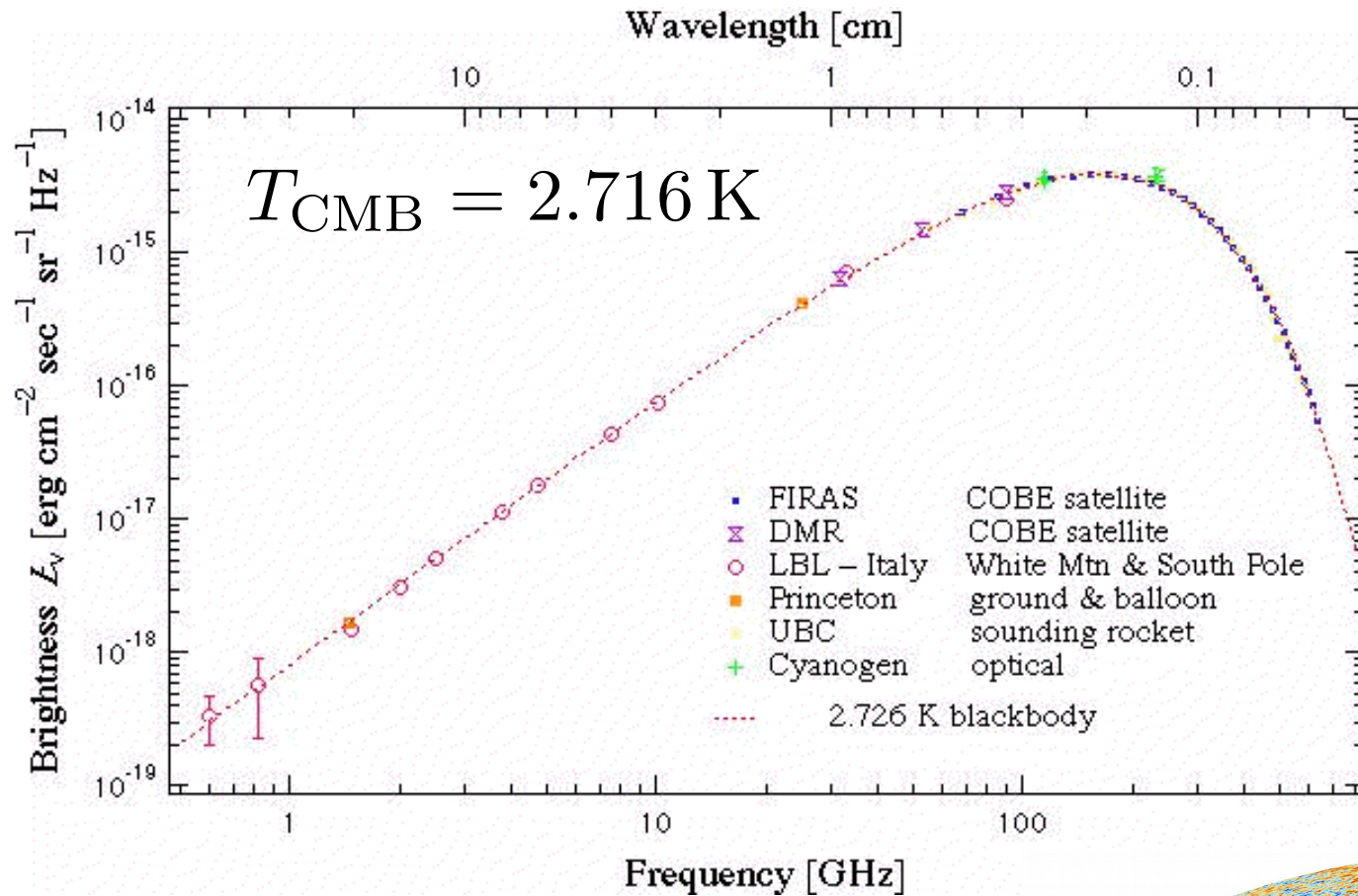
*CMB  
nearly perfect black body  
spectrum*



Anisotropies at the level of  $10^{-5}$  (once dipole is removed)

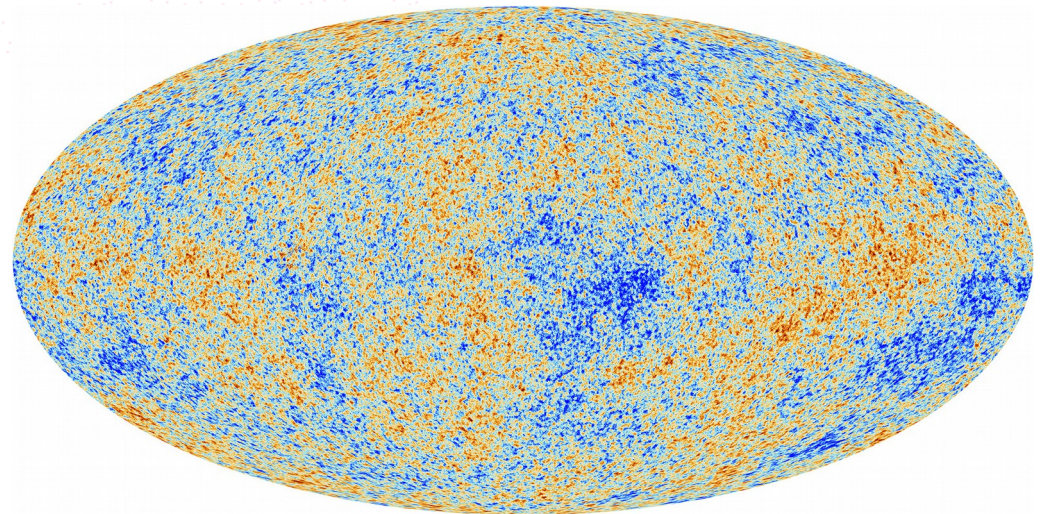


# Hot Big Bang theory : CMB



$$I(\nu, T) \propto \frac{\nu^3}{e^{h\nu/kT} - 1} d\nu$$

$$T \propto a^{-1} = 1 + z$$



# Hot Big Bang theory : CMB

- Cosmic microwave background (CMB)

- black body (BB) spectrum at 2.7 K

→ recombination of  $H^+$  and  $e^-$  around  $T \sim 3000$  K

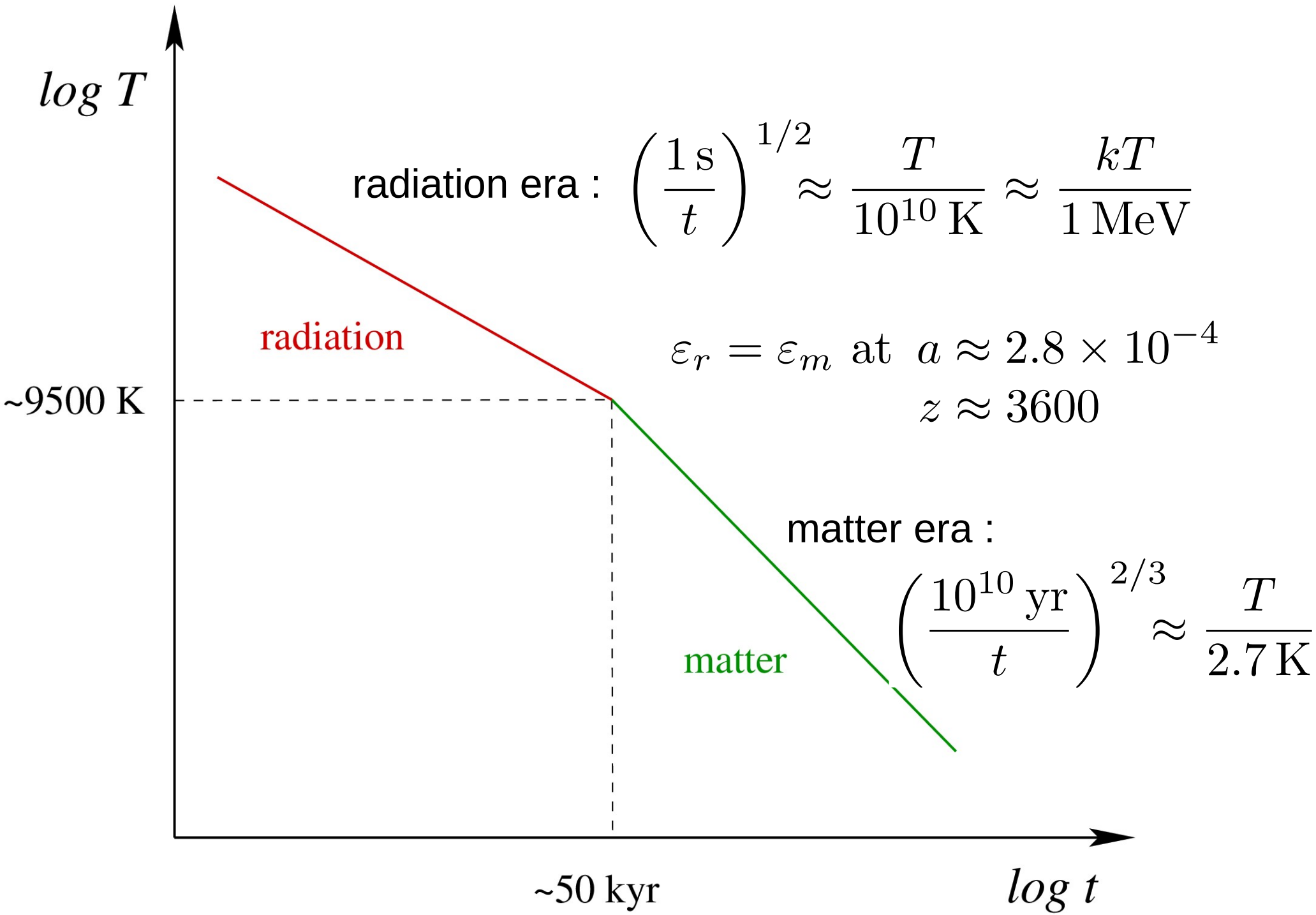
- redshifted by the expansion  $\frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = 1 + z = \frac{1}{a_{\text{emit}}}$

- BB spectrum redshifted is still a BB spectrum !

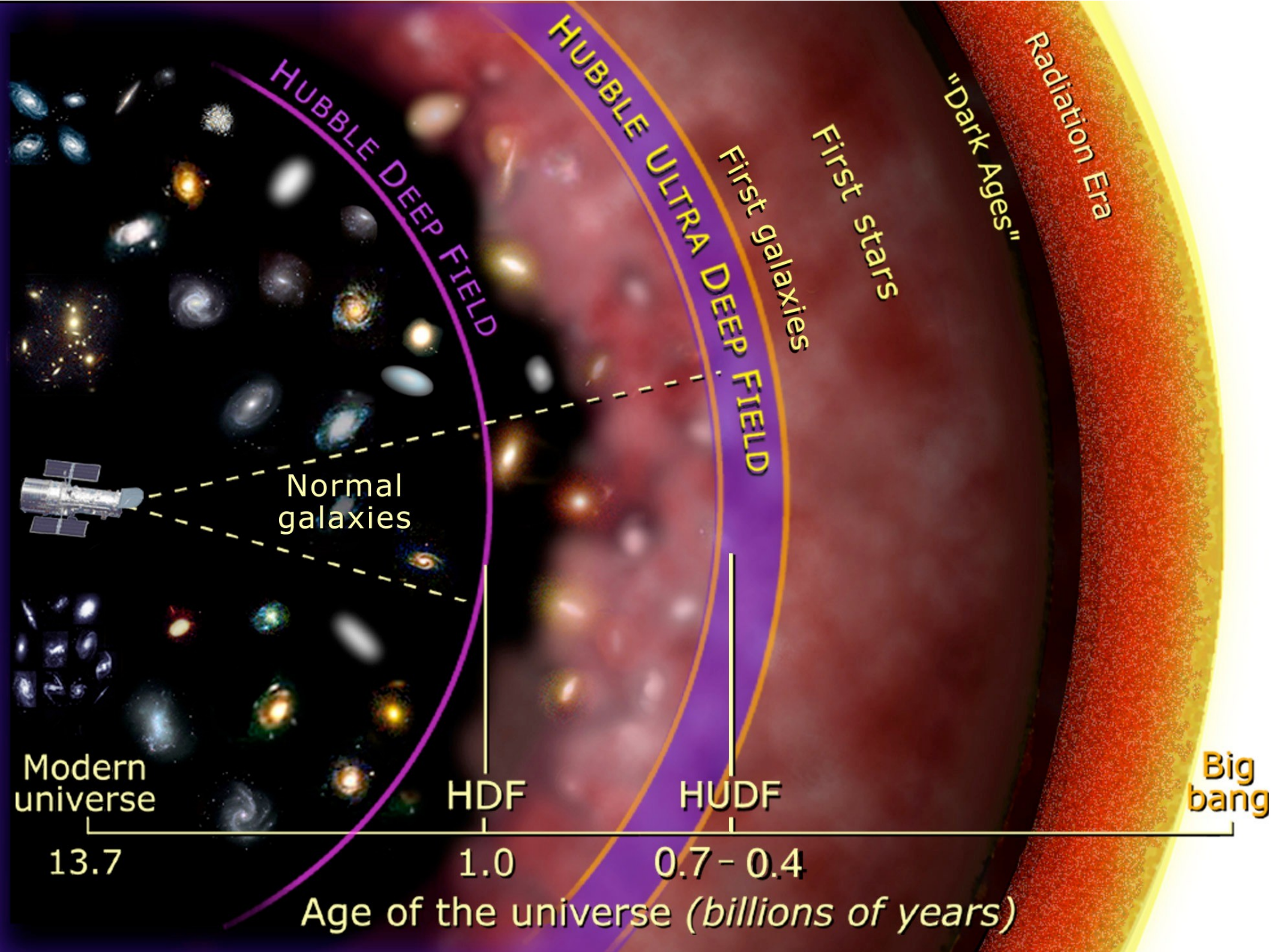
$$I(\nu, T) d\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} d\nu$$

$$z_{\text{CMB}} \approx 1100 \quad T_{\text{CMB},0} = \frac{T_{\text{CMB}}}{1 + z}$$

# Universe : cooling history

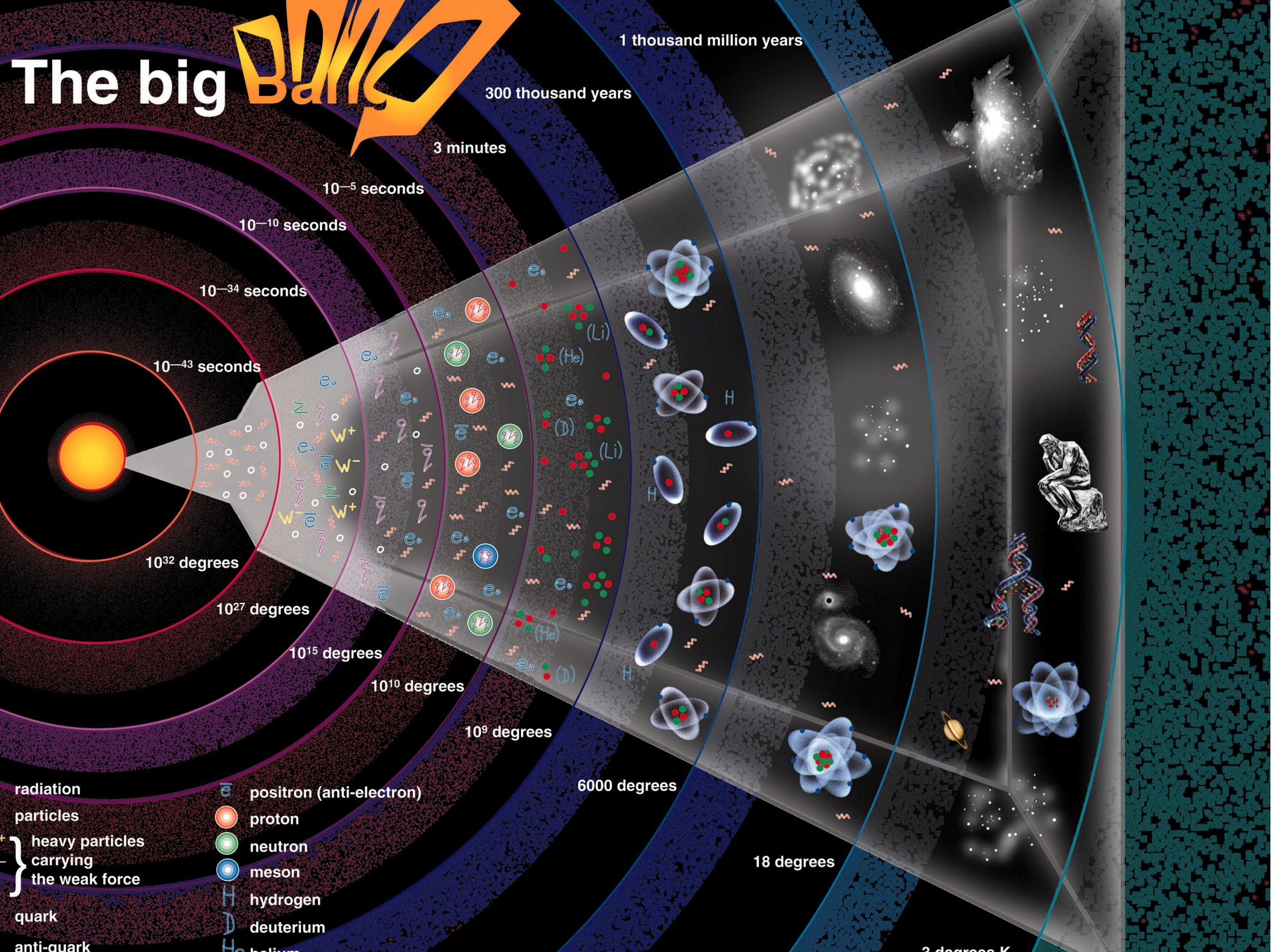








# The big Bang



radiation particles  
 } heavy particles carrying the weak force  
 quark  
 anti-quark

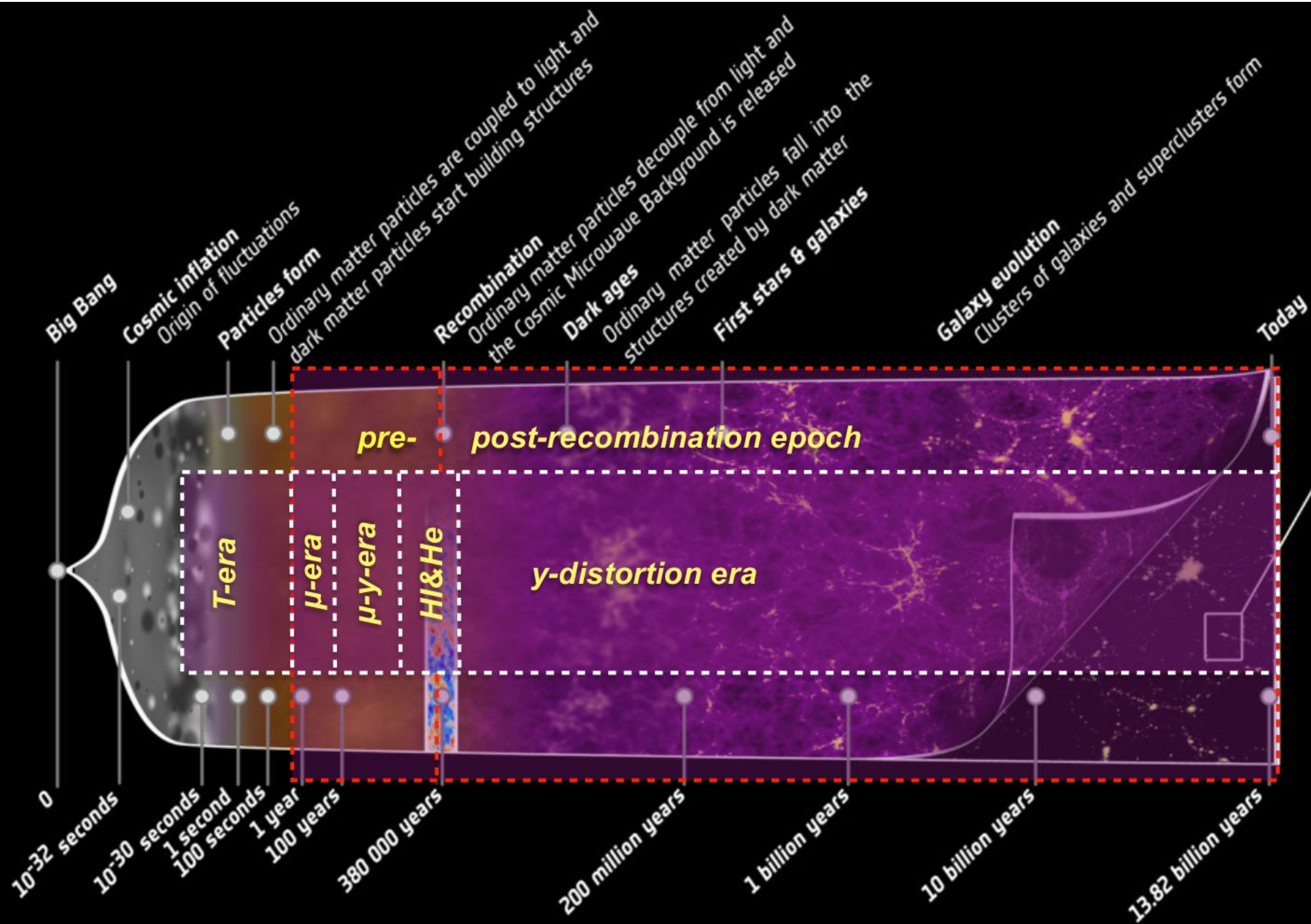
$e^-$  positron (anti-electron)  
 proton  
 neutron  
 meson  
 H hydrogen  
 D deuterium  
 He helium

1 thousand million years  
 300 thousand years  
 3 minutes  
 10<sup>-5</sup> seconds  
 10<sup>-10</sup> seconds  
 10<sup>-34</sup> seconds  
 10<sup>-43</sup> seconds  
 10<sup>32</sup> degrees  
 10<sup>27</sup> degrees  
 10<sup>15</sup> degrees  
 10<sup>10</sup> degrees  
 10<sup>9</sup> degrees  
 6000 degrees  
 18 degrees  
 3 degrees K



# *Universe : cooling history*

Event	$T$	$kT$	$z$	$t$
Quantum gravity	$10^{32}$ K	$10^{19}$ GeV	$10^{32}$	$10^{-43}$ s
Grand unification	$10^{28}$ K	$10^{15}$ GeV	$10^{28}$	$10^{-36}$ s
E-W unification	$10^{15.5}$ K	250 GeV	$10^{15}$	$10^{-12}$ s
Nucleon pairs	$10^{13}$ K	1 GeV	$10^{13}$	$10^{-7}$ s
Neutrino decoupl.	$10^{10}$ K	$\sim 1$ MeV	$10^{10}$	1 s
Nucleosynthesis		10-0.1 MeV		$10^{-1} - 10^3$ s
Pairs e+ e-	$10^{9.7}$ K	0.5 MeV	$10^{9.5}$	4 s
$\varepsilon_r = \varepsilon_m$	9500 K	0.8 eV	3600	50 kyr
Recombination	3000 K	3 eV	1100	380 kyr
Reionization			11 – 4	0.4 – 0.1 Myr
First Galaxies	16 K	$10^{-3}$ eV	5	1 Gyr
$\varepsilon_m = \varepsilon_\Lambda$		$3 \cdot 10^{-4}$ eV	0.3	10 Gyr
Now	2.7 K	$2 \cdot 10^{-4}$ eV	0	13.5 Gyr



# Planck scales : mass, time, length, ...

- Fundamental constants :

$$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.579 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$G = 6.674 \times 10^{-11} \text{ kg}^{-1} \cdot \text{m}^3 \cdot \text{s}^{-2}$$

$$c = 299\,792\,458 \text{ m} \cdot \text{s}^{-1}$$

- « Natural » units :  $\hbar^\alpha c^\beta G^\gamma$

- Planck length  $\ell_{\text{Pl}} = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} \text{ m}$

- Planck time  $t_{\text{Pl}} = \ell_{\text{Pl}}/c = 5.391 \times 10^{-44} \text{ s}$

- Planck mass  $m_{\text{Pl}} = \sqrt{\frac{\hbar c}{G}} = 2.177 \times 10^{-8} \text{ kg}$   
Max particle mass

- Planck temperature  $T_{\text{Pl}} = \frac{m_{\text{Pl}} c^2}{k_{\text{B}}} \simeq 1.417 \times 10^{32} \text{ K}$   
Quantum gravity

# 4.2

## ***Equilibrium distributions***

*Studying the hot universe plasma...*

# *Thermal equilibrium in the early universe*

- Hypotheses :
  - ~ Ideal gas – interactions are **negligible**
  - **Thermal equilibrium** (tiny fraction of collision occurs)
- Equilibrium between radiation (dominating) ↔ matter
$$\gamma + \gamma \rightleftharpoons A + \bar{A} \quad \Gamma$$
- As long as  $\frac{1}{\Gamma} \ll \frac{1}{H}$  *i.e.*  $\Gamma \gg H$
- Particles stay in **thermal equilibrium with the photons...**
  - ... Until they **stop interacting** when  $\Gamma \ll H$
  - ... then they **decouple** (*freeze*)

# Thermal equilibrium : distributions

- Density of states in 6D phase space

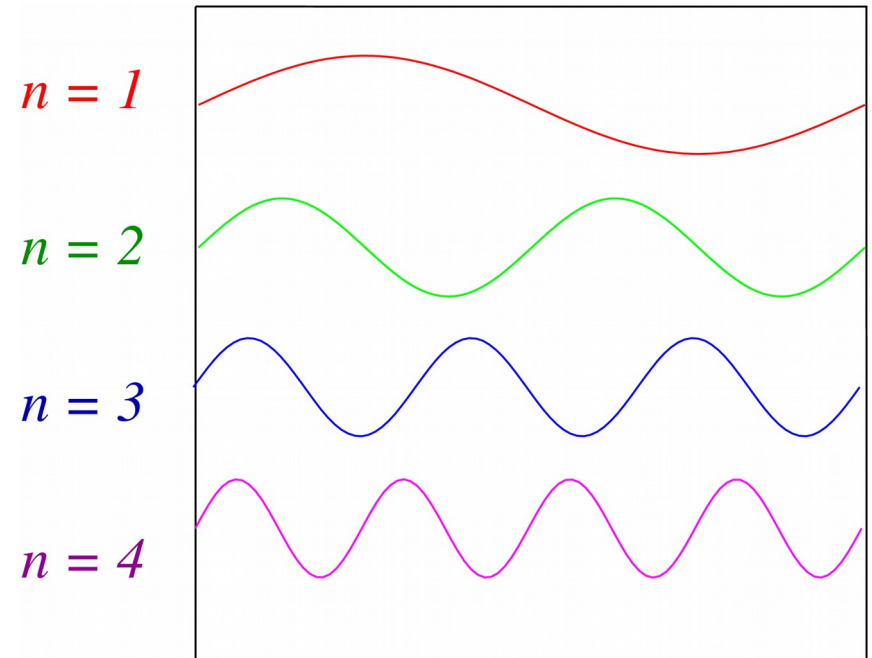
$$\lambda = L/n$$

$$k = \frac{2\pi}{\lambda} = n\Delta k \quad \Delta k = 2\pi/L$$

$$\frac{dn}{d^3\mathbf{k} d^3\mathbf{x}} = \frac{g}{L^3 \Delta k^3} = \frac{g}{(2\pi)^3}$$

$$d^3\mathbf{k} = 4\pi k^2 dk \quad p = \hbar k$$

Waves in a box of size L



- Particle density ( $g$  : degrees of freedom)

$$n = \frac{dn}{d^3\mathbf{x}} = \int \frac{g f(k)}{(2\pi)^3} d^3\mathbf{k} = \frac{g}{(2\pi)^3} \int_0^{+\infty} f(p) 4\pi p^2 dp$$

# Thermal equilibrium : distributions

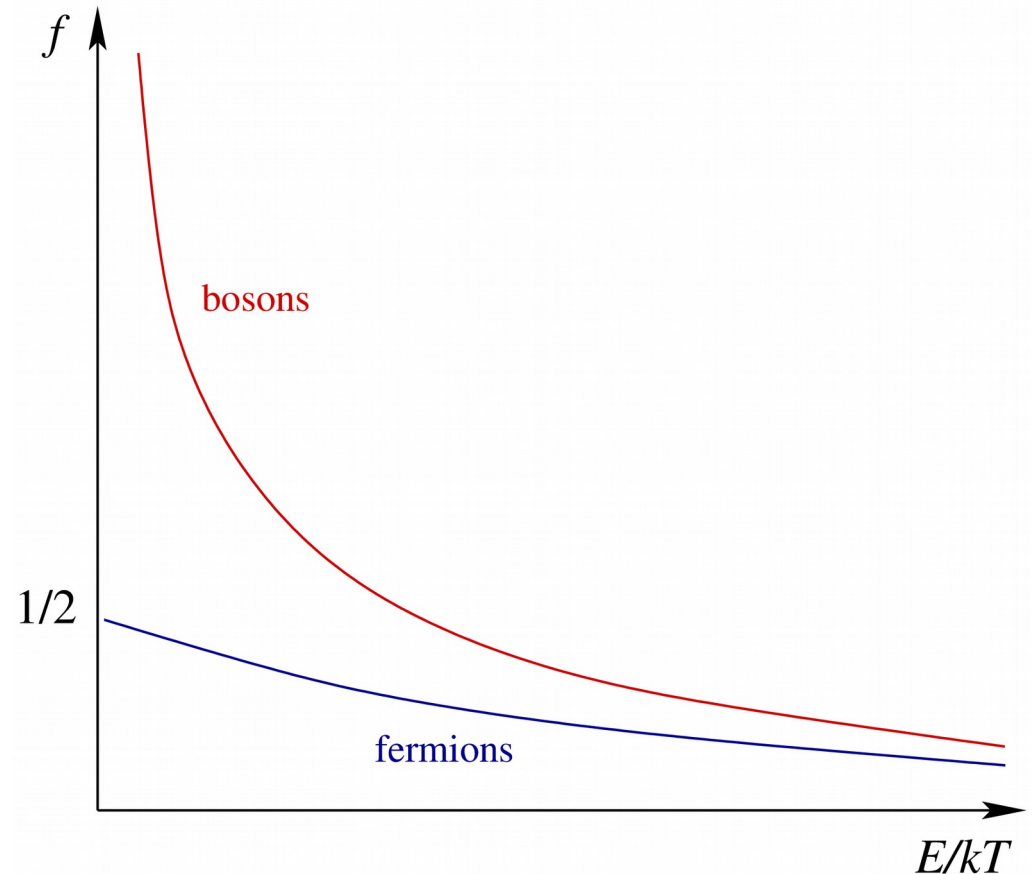
- Occupation density

$$f(\mathbf{p}) = \frac{1}{e^{(E-\mu)/k_B T} \pm 1}$$

+ fermions

- bosons

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$



$$f(\mathbf{p}) = \frac{1}{e^{E/k_B T} \pm 1}$$

$\mu = 0$  photons

$\mu \approx 0$  symmetry particle/antiparticle

# *Thermal equilibrium : distributions*

- Number density  $n = \frac{g}{(2\pi\hbar)^3} \int_0^{+\infty} f(\mathbf{p}) 4\pi p^2 dp$
- Energy density  $\varepsilon = \frac{g}{(2\pi\hbar)^3} \int_0^{+\infty} E(\mathbf{p}) f(\mathbf{p}) 4\pi p^2 dp$
- Pressure  $P = \frac{g}{(2\pi\hbar)^3} \int_0^{+\infty} \frac{p^2}{3E(\mathbf{p})} f(\mathbf{p}) 4\pi p^2 dp$
- Entropy  $S(T) = \frac{E + PV}{T}$        $s = \frac{S}{V} = \frac{\varepsilon + P}{T}$
- Entropy is conserved

$$dS = 0 \quad sa^3 = \text{cste}$$



# *Relativistic / non relativistic particles*

*Non relativistic matter  
Cold matter*

$$m > 0 \quad p \ll mc$$

$$E = \gamma mc^2 \approx mc^2$$

$$\varepsilon_m \approx Nmc^2 a^3 \propto a^{-3}$$

*Radiation, relativistic particles  
Hot matter*

$$m = 0 \quad \text{or} \quad m > 0 \quad p \gg mc$$

$$E = h\nu = hc/\lambda \propto a^{-1}$$

$$\varepsilon_r = Nh\nu a^{-3} \propto a^{-4}$$

# Relativistic limit

$$k_B T \gg mc^2 \quad E \approx pc$$

- Number density

$$n = \frac{g}{(2\pi\hbar)^3} \int_0^{+\infty} \frac{4\pi p^2 dp}{e^{pc/kT} \pm 1} = \frac{4\pi g}{(2\pi\hbar)^3} \left(\frac{k_B T}{c}\right)^3 \int_0^{+\infty} \frac{y^2 dy}{e^y \pm 1}$$

## Energy density

$$\varepsilon = \frac{g}{(2\pi\hbar)^3} \int_0^{+\infty} \frac{4\pi p^3 c dp}{e^{pc/kT} \pm 1} = \frac{4\pi g}{(2\pi\hbar)^3} \frac{(k_B T)^4}{c^3} \int_0^{+\infty} \frac{y^3 dy}{e^y \pm 1}$$

# *Useful Integrals and formulas...*

For bosons  $I_n = \int_0^{+\infty} \frac{y^n dy}{e^y - 1} = \Gamma(n + 1)\zeta(n + 1)$

For fermions  $J_n = \int_0^{+\infty} \frac{y^n dy}{e^y + 1} = \left(1 - \frac{1}{2^n}\right) \Gamma(n + 1)\zeta(n + 1)$

Gamma function

$$\Gamma(n + 1) = n!$$

Riemann zeta function

$$\zeta(2) = \frac{\pi^2}{6} \quad \zeta(3) \simeq 1.202 \quad \zeta(4) = \frac{\pi^4}{90}$$

# Relativistic limit

- Number density

$$n = \frac{\zeta(3)}{\pi^2} \frac{1}{(\hbar c)^3} g (k_B T)^3 \times \begin{cases} 3/4 & \text{fermions} \\ 1 & \text{bosons} \end{cases} \quad n \propto g T^3$$

- Energy density

$$\varepsilon = \frac{\pi^2}{30} g \frac{(k_B T)^4}{(\hbar c)^3} \times \begin{cases} 7/8 & \text{fermions} \\ 1 & \text{bosons} \end{cases} \quad \varepsilon \propto g T^4$$

- Pressure  $P = \frac{\varepsilon}{3}$   $w = 1/3$

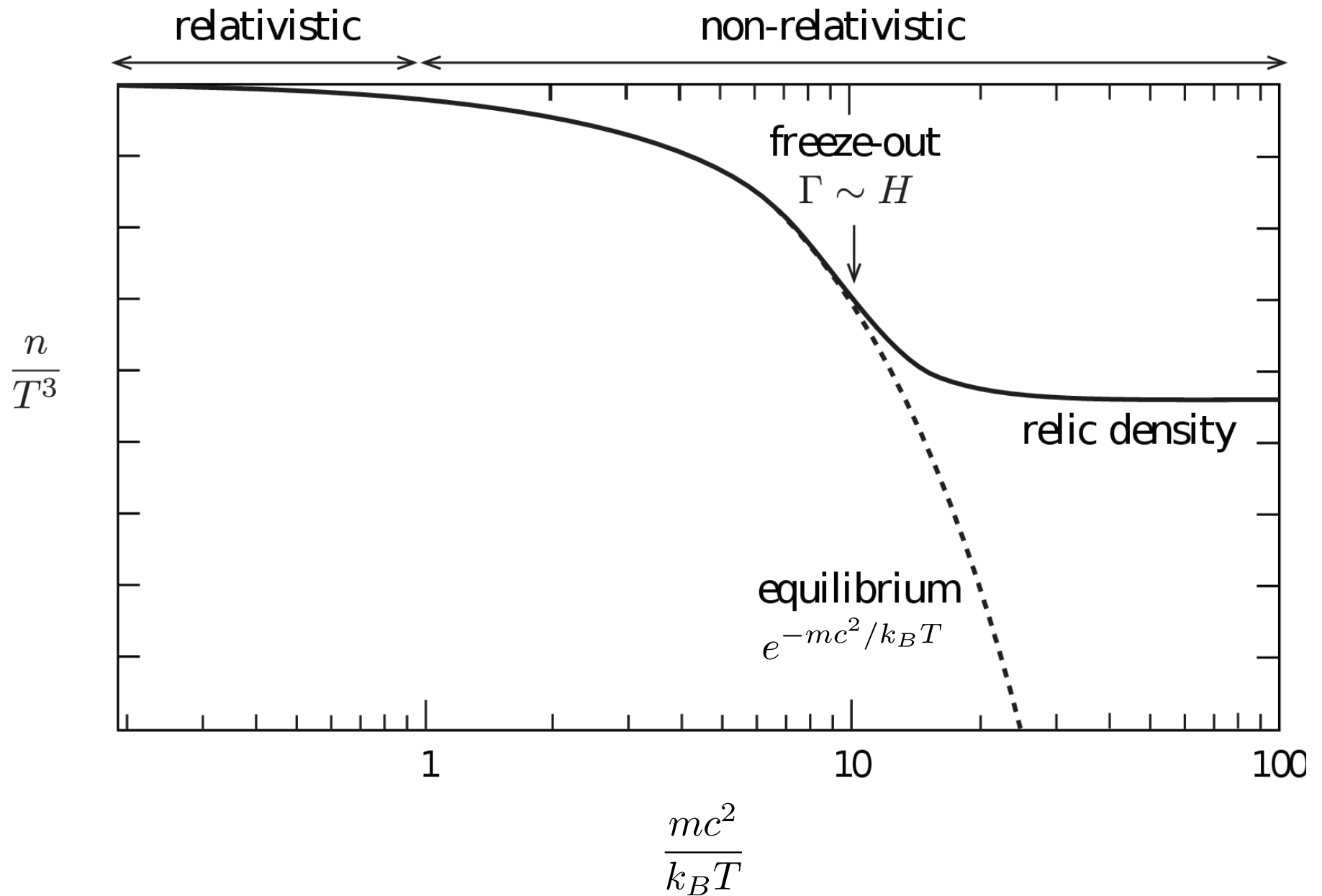
- Entropy  $s = \frac{\varepsilon + P}{T} = \frac{4}{3} \frac{\varepsilon}{T} = 3.602 k_B n \times \begin{cases} 7/6 & \text{fermions} \\ 1 & \text{bosons} \end{cases}$

$$s \propto g T^3 \quad s \propto n$$

# Relativistic and non relativistic limits

	$kT \gg mc^2$		$kT \ll mc^2$
	Bosons	Fermions	Non relativistic gas
$n$	$\frac{\zeta(3)}{\pi^2} g_B \frac{(k_B T)^3}{(\hbar c)^3}$	$\frac{3}{4} \frac{\zeta(3)}{\pi^2} g_B \frac{(k_B T)^3}{(\hbar c)^3}$	$g \left( \frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} e^{\frac{-mc^2}{k_B T}}$
$\varepsilon$	$\frac{\pi^2}{30} g_B \frac{(k_B T)^4}{(\hbar c)^3}$	$\frac{7}{8} \frac{\pi^2}{30} g_F \frac{(k_B T)^4}{(\hbar c)^3}$	$n \left( mc^2 + \frac{3}{2} k_B T \right)$
$P$	$\frac{\varepsilon}{3}$	$\frac{\varepsilon}{3}$	$n k_B T$
$s$	$\frac{4}{3} \frac{\varepsilon}{T}$	$\frac{4}{3} \frac{\varepsilon}{T}$	$\frac{n}{T} \left( mc^2 + \frac{5}{2} k_B T \right)$

*Number density of non relativistic particles exponentially suppressed, until they decouple from thermal bath...*



*Number density of non relativistic particle exponentially suppressed, until they decouple from thermal bath...*

# Total energy density

$$\varepsilon(T) = \varepsilon_{\text{rel.}} + \varepsilon_{\text{n.r.}} = \varepsilon_{\text{rel.}}^{\text{th.}} + \varepsilon_{\text{n.r.}}^{\text{th.}} + \varepsilon_{\text{rel.}}^{\text{dec.}} + \varepsilon_{\text{n.r.}}^{\text{dec.}}$$

$$\varepsilon_{\text{rel.}} = \sum_{B,i} g_{B,i} \frac{\pi^2}{30} \frac{(k_B T_i)^4}{(\hbar c)^3} + \frac{7}{8} \sum_{F,i} g_{F,i} \frac{\pi^2}{30} \frac{(k_B T_j)^4}{(\hbar c)^3}$$

$$\varepsilon_{\text{rel.}} = g_* \frac{\pi^2}{30} \frac{k_B^4}{\hbar^3 c^3} T^4$$

Where

$$g_*(T) = \sum_{B,i} g_{B,i} \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{F,j} g_{F,j} \left( \frac{T_j}{T} \right)^4$$

For particles still at equilibrium with the photons (« th »),  $T_i = T_j = T$

$$g_*^{\text{th}}(T) = \sum_{B,i} g_{B,i} + \frac{7}{8} \sum_{F,j} g_{F,j}$$

Once a particle population is non relativistic, it is removed from  $g_*$

# Entropy

$$s_{\text{rel.}} = \sum_{B,i} g_{B,i} \frac{2\pi^2}{45} \frac{k_B^4 T_i^3}{(\hbar c)^3} + \frac{7}{8} \sum_{F,i} g_{F,i} \frac{2\pi^2}{45} \frac{k_B^4 T_j^3}{(\hbar c)^3}$$

$$s_{\text{rel.}} = \frac{2\pi^2}{45} \frac{k_B^4}{(\hbar c)^3} g_{*,s} T^3$$

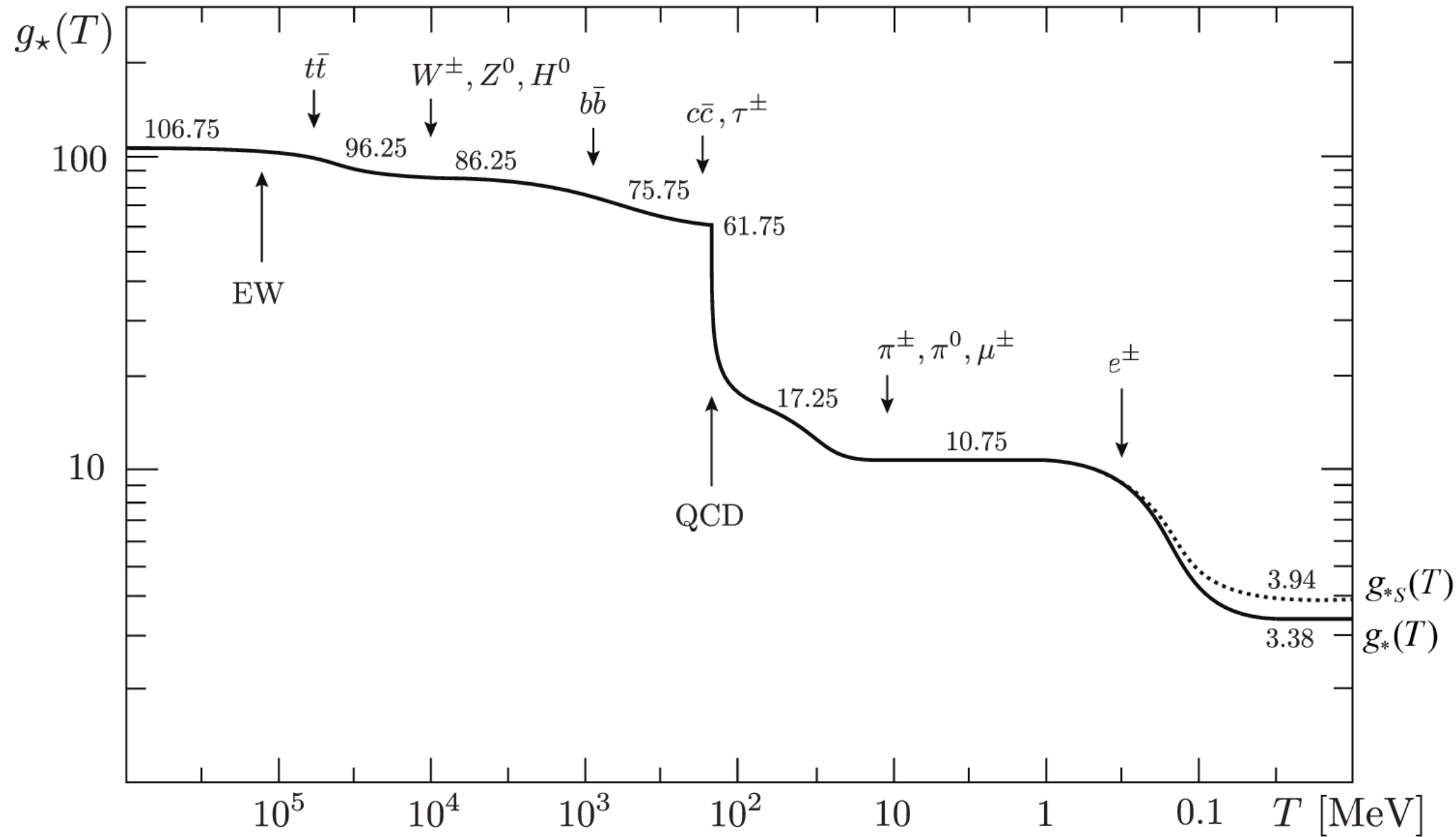
Where  $g_{*,s}(T) = \sum_{B,i} g_{B,i} \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{F,j} g_{F,j} \left(\frac{T_j}{T}\right)^3 \neq g_*(T)$

For particles still at equilibrium with the photons (« th »),  $T_i = T_j = T$

$$g_{*,s}^{\text{th}}(T) = \sum_{B,i} g_{B,i} + \frac{7}{8} \sum_{F,j} g_{F,j} = g_*^{\text{th}}(T)$$

When a particle species decouples, its entropy is transferred to the heat bath... and  $g_{*,s}$  drops. Total entropy is conserved.





$$g_*, g_{*,s}$$

When  $k_B T \gg 175 \text{ GeV}$

All SM particles are relativistic :

Fermions :

$$g_F = 12 \times 6 + 6 \times 2 + 3 \times 2 = 90$$

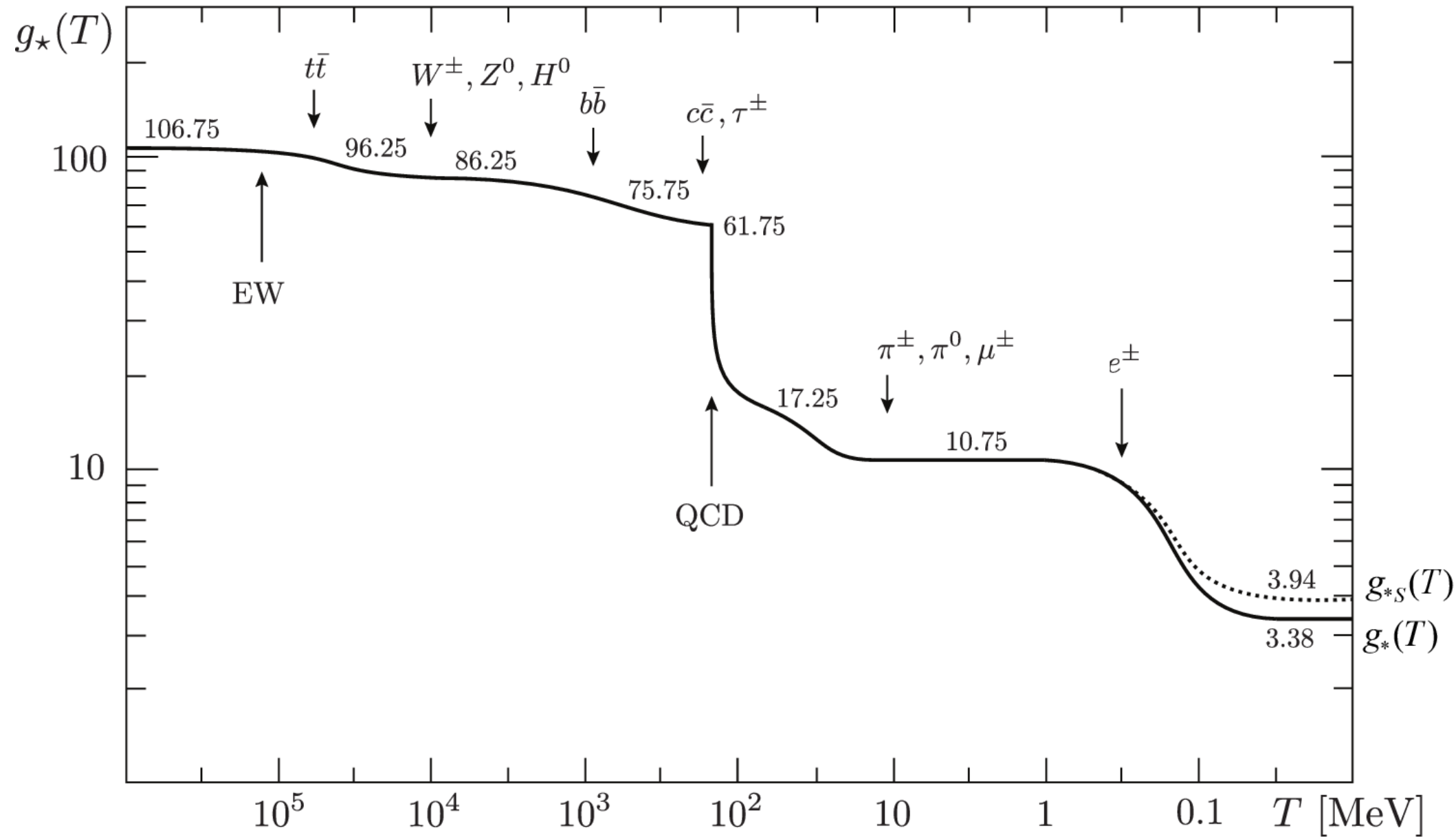
Bosons :

$$g_B = 8 \times 3 + 2 + 3 \times 3 + 1 = 28$$

$$g_* = g_{*,s} = 28 + \frac{7}{8} \times 90$$

$$g_* = g_{*,s} = 106.75$$

type		mass	spin	$g$
quarks	$t, \bar{t}$	173 GeV	$\frac{1}{2}$	$2 \cdot 2 \cdot 3 = 12$
	$b, \bar{b}$	4 GeV		
	$c, \bar{c}$	1 GeV		
	$s, \bar{s}$	100 MeV		
	$d, \bar{d}$	5 MeV		
	$u, \bar{u}$	2 MeV		
gluons	$g_i$	0	1	$8 \cdot 2 = 16$
leptons	$\tau^\pm$	1777 MeV	$\frac{1}{2}$	$2 \cdot 2 = 4$
	$\mu^\pm$	106 MeV		
	$e^\pm$	511 keV		
	$\nu_\tau, \bar{\nu}_\tau$	$< 0.6 \text{ eV}$	$\frac{1}{2}$	$2 \cdot 1 = 2$
	$\nu_\mu, \bar{\nu}_\mu$	$< 0.6 \text{ eV}$		
	$\nu_e, \bar{\nu}_e$	$< 0.6 \text{ eV}$		
gauge bosons	$W^+$	80 GeV	1	3
	$W^-$	80 GeV		
	$Z^0$	91 GeV		
	$\gamma$	0		
Higgs boson	$H^0$	125 GeV	0	1



# Decoupled species

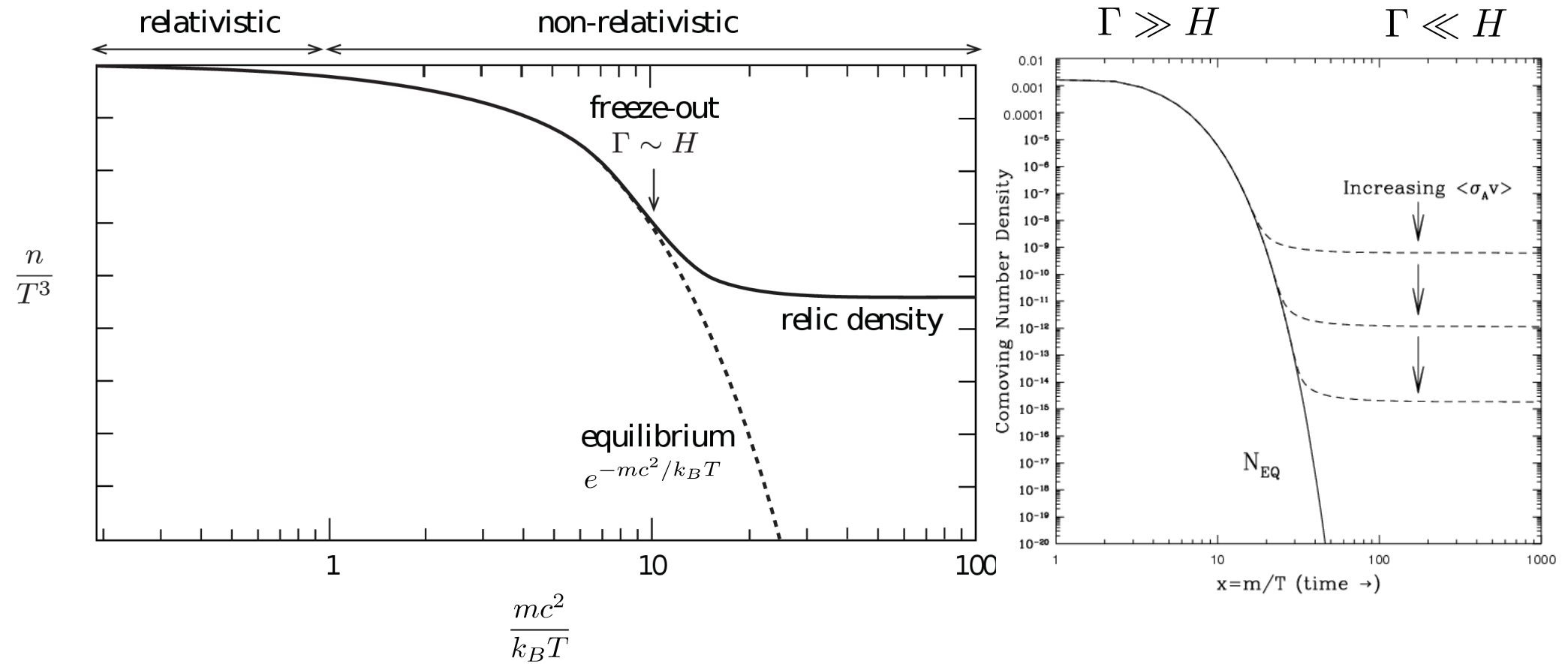
- Before decoupling → **phase space distribution** (prev.)
- Decoupling when  $\Gamma \approx H$
- After decoupling : **relic density** (*frozen*)
- Relativistic species (neutrinos, ...):

$$f(p) = \frac{1}{e^{E/k_B T_{\text{eff}}} \pm 1} \quad T_{\text{eff}} = T_{\text{dec.}} \frac{a_{\text{dec.}}}{a(t)} \propto a^{-1}$$

- Non relativistic species :

particles per unit of comobile volume conserved  
particles dilute...

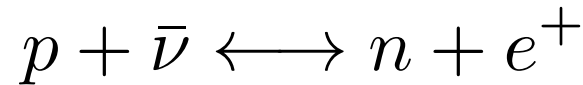
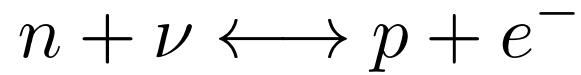
$$n \propto a^{-3} \quad T_{\text{eff}} = T_{\text{dec.}} \left( \frac{a_{\text{dec.}}}{a(t)} \right)^2 \propto a^{-2}$$



*The remaining amount (relic density) depends on the desintegration cross-section.*

# Neutrino decoupling

Equilibrium with heat bath :



$$\frac{\Gamma_\nu}{H} \approx 5.4 \frac{G_F^2}{M_{\text{Pl}}} T^3$$

Happens when  $\Gamma_\nu \approx H$

*i.e.*  $k_B T_\nu^{\text{dec.}} \approx 0.8 \text{ MeV}$

Freeze the neutron / proton ratio

$$\frac{n_n(T)}{n_p(T)} = e^{-(m_n - m_p)c^2 / k_B T}$$

$$\frac{n_n}{n_p} \approx 0.34$$

Entropy conservation :  $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$

# Annihilation pairs $e^+ e^-$

It occurs around  $k_B T \approx mc^2 = 0.511 \text{ MeV}$

$$\text{At time } \frac{t}{1 \text{ s}} \approx \left( \frac{1 \text{ MeV}}{k_B T} \right)^2 \approx \left( \frac{1}{0.511 \text{ MeV}} \right)^2 \approx 4 \quad t \approx 4 \text{ s}$$

$$\text{Entropy conserved : } s(a_1)a_1^3 = s(a_2)a_2^3$$

Before (1):  $\gamma, e^+, e^-, \nu, \bar{\nu}$  (neutrinos already decoupled)

After (2):  $\gamma$  (relic  $e^-$ , neutrinos already decoupled)

$$g_{*,s}(a_1)T_1^3 a_1^3 = g_{*,s}(a_2)T_2^3 a_2^3 \quad T_2 = \left( \frac{11}{4} \right)^{1/3} T_1 \frac{a_1}{a_2}$$

$$T_2 = T_\gamma(a_2) \quad T_1 \frac{a_1}{a_2} = T_\nu(a_2) \quad T_\nu = \left( \frac{4}{11} \right)^{1/3} T_\gamma \quad T_\nu \approx 1.95 \text{ K}$$

# 4.3

## ***Primordial nucleosynthesis***

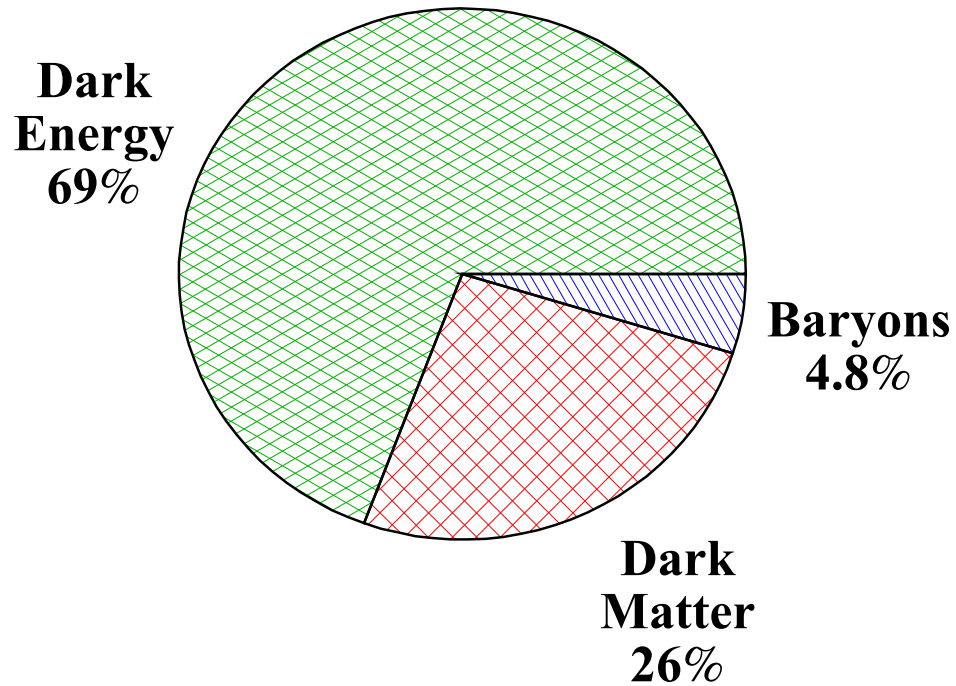
*How the first nuclei appeared...*



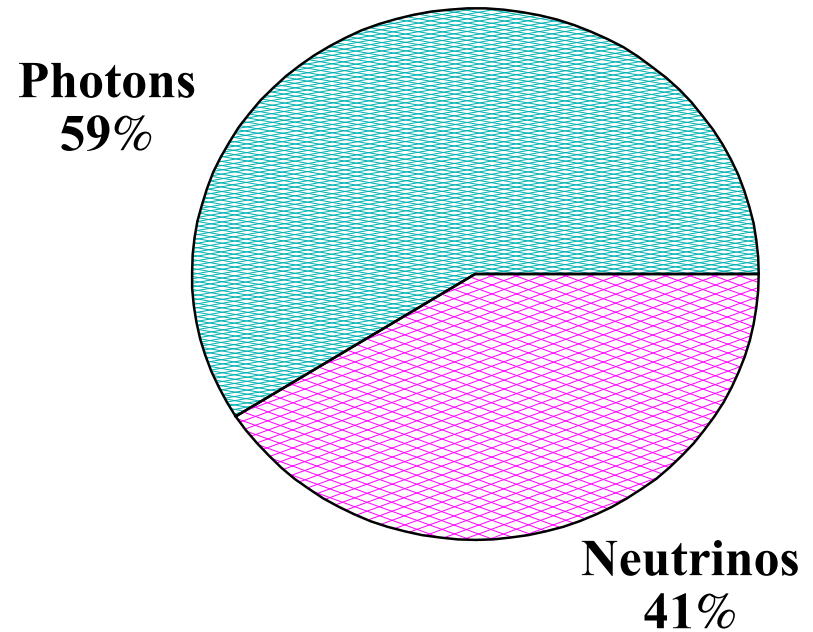
# Big Bang nucleosynthesis

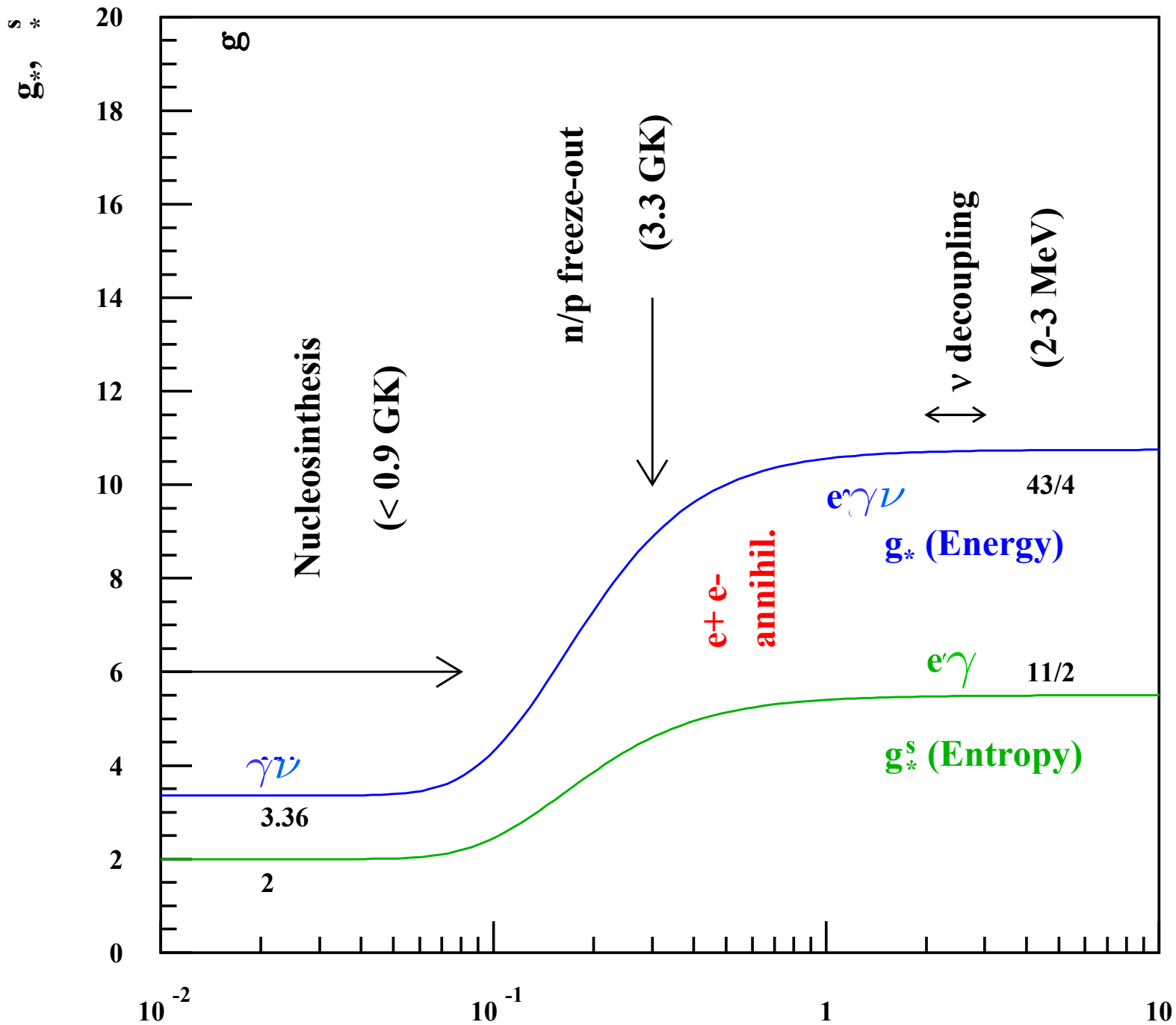
Era dominated by radiation

**Now  $z=0$**



**BBN  $z=10^8$**





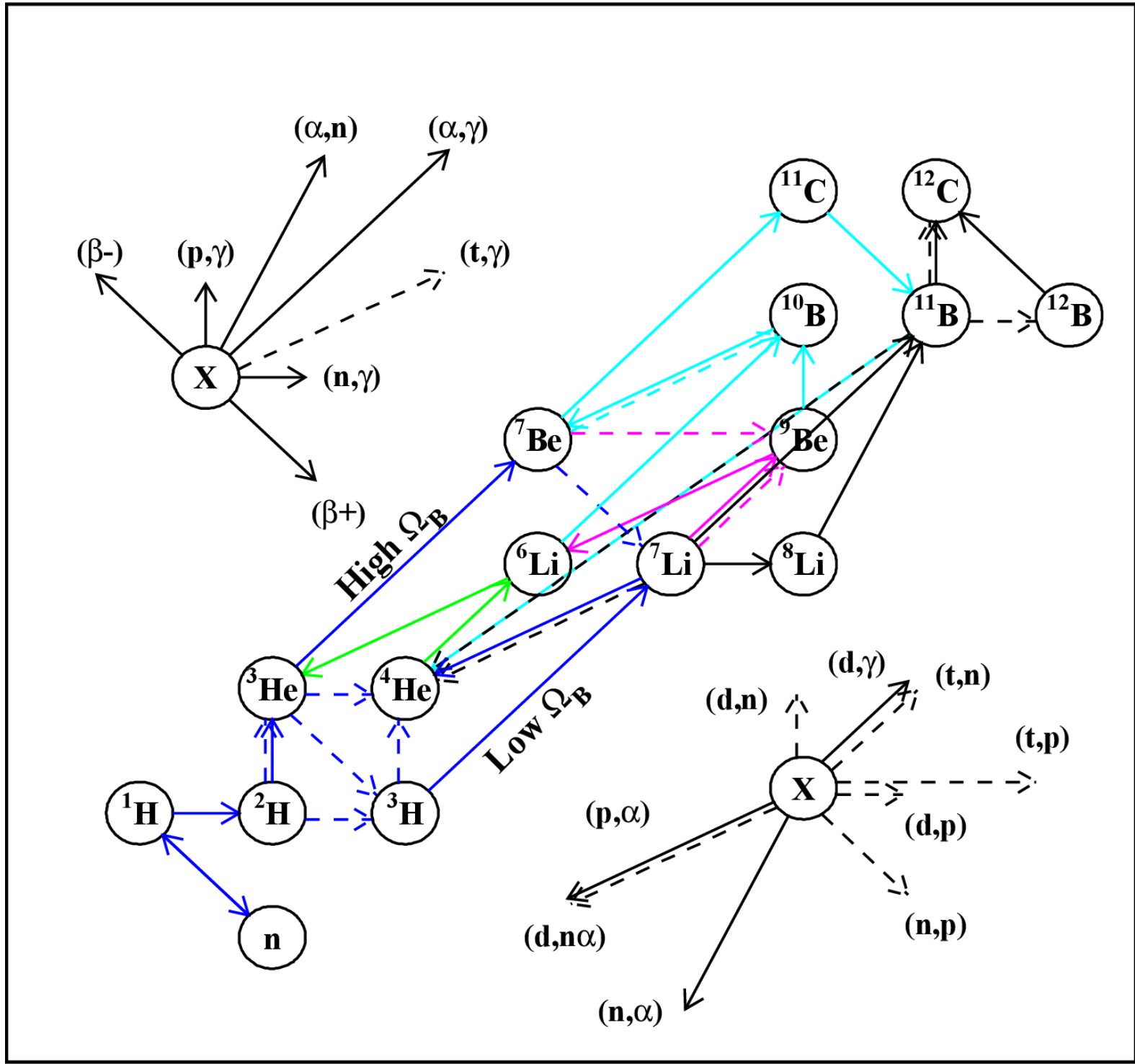
# Big Bang nucleosynthesis

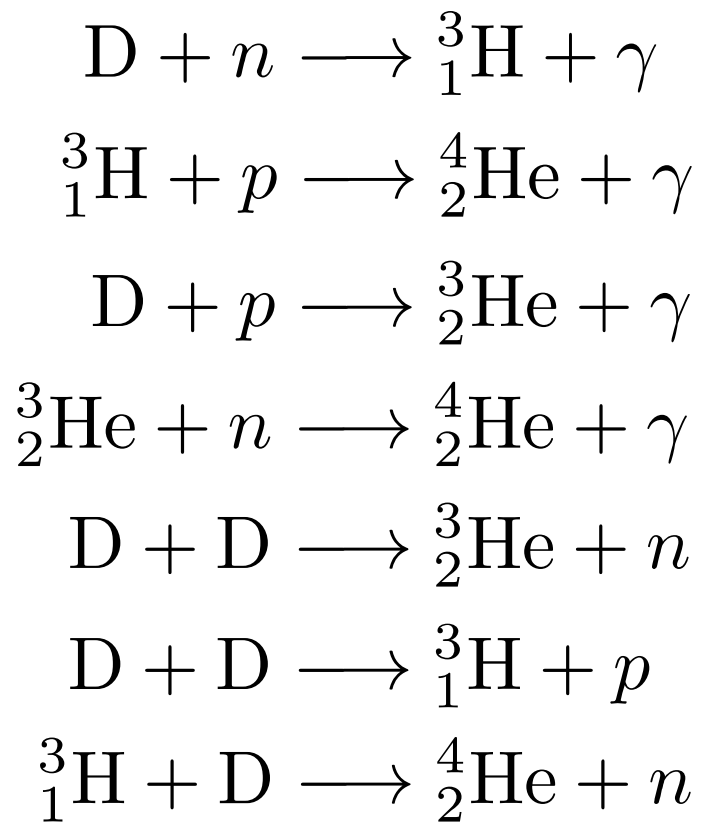
- Critical parameter : density of baryons

$$\Omega_{b,0} = \varepsilon_{b,0}/\varepsilon_{c,0} \quad n_b \approx \frac{\Omega_{b,0}\varepsilon_{c,0}}{m_p}$$

$$\eta = \frac{n_b}{n_\gamma} \simeq 6 \times 10^{-10}$$

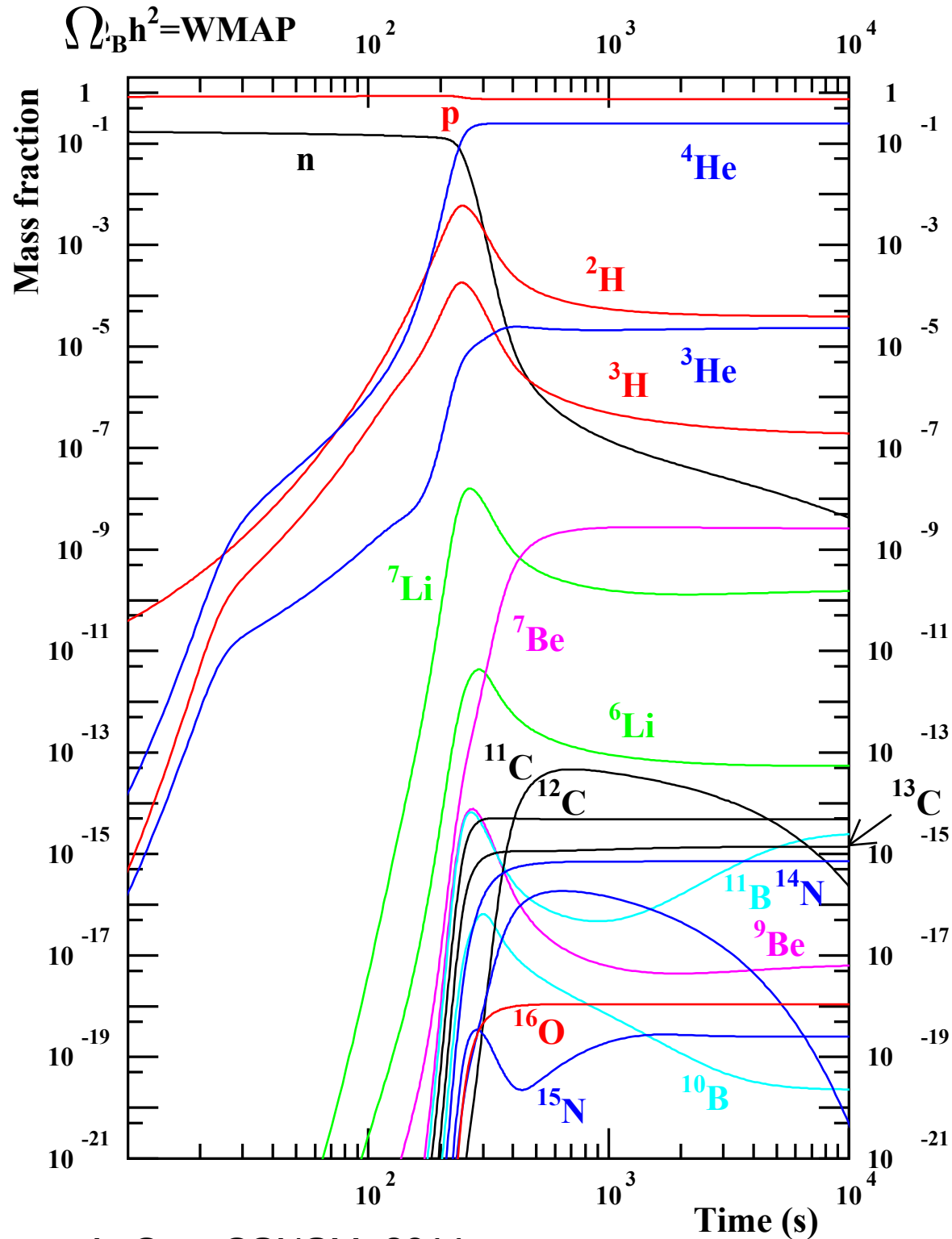
Time < 1 s	$p + e^- \longleftrightarrow n + \nu_e$ $n + e^+ \longleftrightarrow p + \bar{\nu}_e$	Neutron/proton freeze-out
1 – 100 s	$n \longrightarrow p + e^- + \bar{\nu}_e$	Neutrons decay $\tau \approx 880$ s
100 – 200 s	$p + n \longleftrightarrow \text{D} + \gamma$	Deuterium formed Allows neutrons to survive
200 – 1000 s	${}^3\text{H}, {}^3\text{He}, {}^4\text{He}; \dots$	Deuterium burned to produce next elements





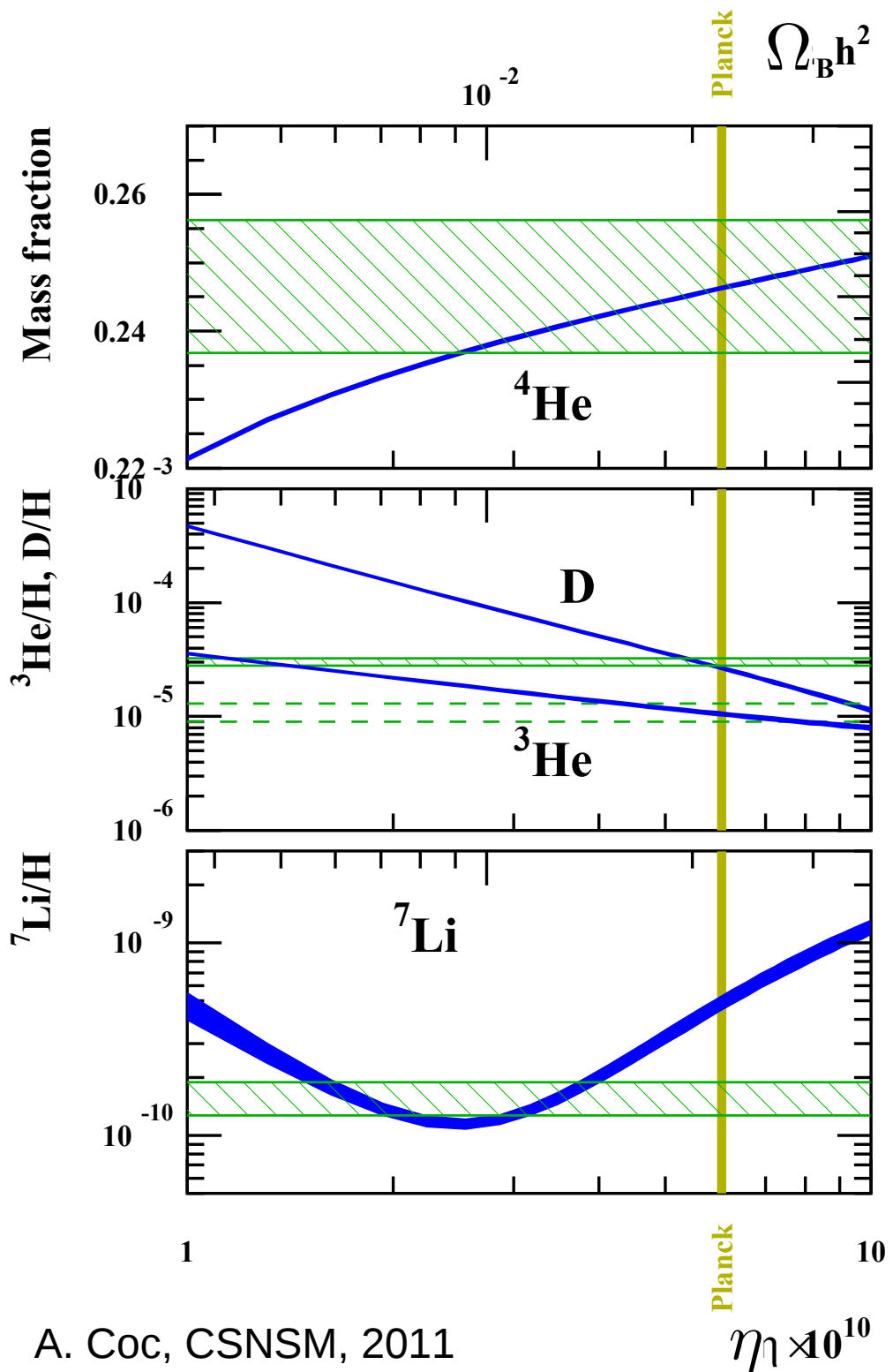
*Etc...*

$$Y_P({}^4_2\text{He}) \approx 0.25$$



$$\eta = \frac{n_b}{n_\gamma} \simeq 6 \times 10^{-10}$$

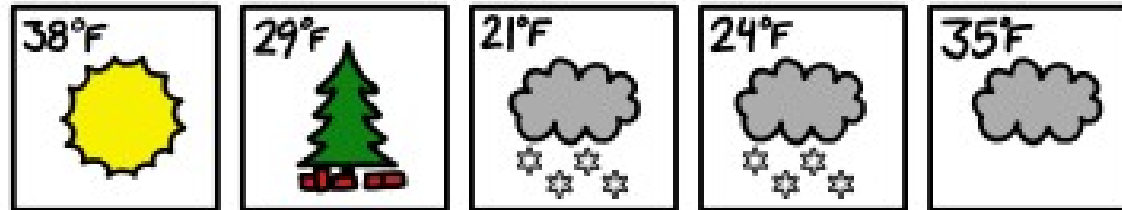
$$\Omega_{b,0} = \varepsilon_{b,0} / \varepsilon_{c,0}$$



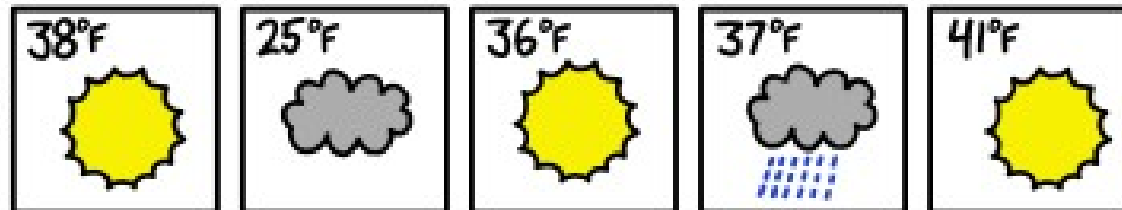
YOUR 5-DAY FORECAST



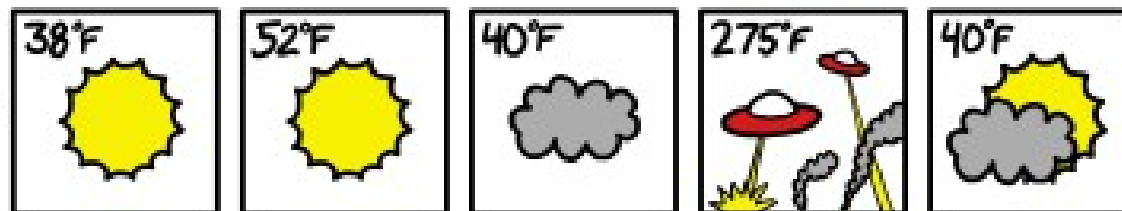
YOUR 5-MONTH FORECAST



YOUR 5-YEAR FORECAST



YOUR 5-MILLION-YEAR FORECAST



YOUR 5-BILLION-YEAR FORECAST



YOUR 5-TRILLION-YEAR FORECAST

